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Studies have been made of fluxon dynamics, instabilities, and noise in long overlap type Josephson junctions made out of Niobium-Nitride and Niobium. Noise and fluctuations measurements were performed in the voltage state of the junctions biased in a magnetic field. We observed telegraph noise due to fluxon fluctuations between two energy states determined by the geometry of the junction and the external magnetic field bias. The studies covered lifetimes of these states, metastability, thermal and quantum mechanical tunneling, and effects of dissipation. At certain bias points of the junction we found chaotic behavior which was preceded by period-doubling bifurcation. Also, negative resistance regions were observed; they can be used for amplification. These observations were supported by computer modeling using a perturbed sine-Gordon equation. Fluxon motion in long Josephson junctions provides an excellent system for studies of non-linear phenomena and it has a variety of applications to superconducting electronics. *Keywords:*

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## ABSTRACT

This report presents experiments performed on long Josephson junctions where studies were made of fluxon dynamics, instabilities, and noise. The junctions were all NbN or Nb of the overlap type. Noise and fluctuation measurements were performed in the voltage state of the junctions where biased in a magnetic field. We observed telegraph noise due to fluxon fluctuations between 2 energy states determined by the geometry of the junction and the external magnetic field bias. The studies covered lifetimes of such states, metastability, thermal and quantum mechanical tunneling, and effects of dissipation. At certain bias points of the junction we found chaotic behavior which was preceded by period-doubling bifurcation. We also observed negative resistance regions where amplification could take place. These observations were supported by computer modeling of the system using a perturbed sine-Gordon equation. The fluxon motion in a long junction provides an excellent simple system for studying non-linear phenomena and it has a variety of interesting applications.

## 1. INTRODUCTION

There has been a great deal of elegant and very useful superconducting electronics<sup>1,2</sup> developed with the unit called SQUID; it uses small Josephson junctions. This has been a popular approach because a small Josephson junction is relatively simple to make; moreover, although a small junction by itself has no gain, the incorporation of 2 or more such junctions in an interferometer (the SQUID) has gain which is necessary for circuit development. In this research a different approach has been taken. Instead of dealing with flux quantization within the SQUID loop, it is possible to consider flux quantization inside a Josephson junction when it is long enough<sup>3-5</sup>. Such flux, the fluxon, can be considered as a basic element in superconducting electronics and hence a new class of interesting applications can be developed. This project deals with the behavior of quantized current vortices, fluxons, in long Josephson junctions. The research presented here consists of investigating the fundamental aspects of fluxons in junctions and related phenomena, keeping an eye for possible applications.

### 1.1 Objectives

The goals of this research are to study

- a. noise and fluctuations in long Josephson junctions
- b. the dynamics of fluxon motion
- c. effects of disorder inside the junction
- d. instabilities and non-linear effects of fluxon motion
- e. fluxon tunneling: thermal and quantum mechanics aspects
- f. effects of dissipation on fluxon motion.

### 1.2 Fluxons in Long Josephson Junctions

In a bulk superconductor, the interior of the sample is shielded from the external magnetic fields by induced currents flowing within the London penetration depth  $\lambda_L$  of the surface. This parameter is inversely proportional to the density of superconducting electrons and it has a fixed value for a given superconductor. In a Josephson junction, similar behavior can

occur and shielding takes place within a characteristic length  $\lambda_J$ , the Josephson penetration depth. It depends inversely on the current density  $J_c$  and it is given by

$$\lambda_J = \left[ \left( \frac{\phi_0}{2\pi} \right) \frac{1}{\mu_0 d J_c} \right]^{1/2} \quad (1)$$

where  $d$  is the magnetic thickness (the barrier thickness  $t$  plus  $2\lambda_L$ ) and  $\phi_0$  the flux quantum. The parameter  $\lambda_J$  can be changed by modifying the critical current density of the junction.

A Josephson junction is considered long when one of its dimensions  $L$  is larger than  $\lambda_J$ . In that case, penetration of magnetic flux occurs above a critical field  $B_{c_1}$  which then produces magnetic flux quanta inside the junction. Such quanta, known as fluxons, have dimensions of the order of  $\pi \lambda_J$ . The free energy of a fluxon is given by

$$E = \left( \frac{\hbar J_c}{2e} \right) \pi \lambda_J \quad (2)$$

Thus the length  $\lambda_J$  sets the scale of the device and also the energies involved. Neglecting demagnetizing effects, the formation of fluxons inside the junction<sup>6</sup> occurs when the external magnetic field or electrical currents satisfy the condition

$$B_{c_1} = \frac{4}{\pi} J_c \lambda_J \quad (3)$$

In the presence of an electric current, such as a bias current through the junction, there will be a Lorentz force on the fluxons causing them to move. However, due to friction, a constant fluxon

velocity is achieved in a distance of  $\sim \lambda_J \beta_c^{1/2}$  (the McCumber parameter  $\beta_c = \left(\frac{2e}{\hbar}\right) I_c R_N^2 C$  is a measure of the damping of the junction and of hysteresis behavior due to the capacitance  $C$ ; for example, when  $\beta_c \gg 1$  the junction has low damping and there is considerable hysteresis). The dissipation due to friction comes from the quasi particles present and from the high frequency surface losses in the electrodes. Figure 1 shows fluxons in a long junction.

The long junction can act as a cavity with its own resonant frequency and one can consider the junction as being coupled to this cavity in the following way. The junction has Josephson oscillations given by  $\omega = 2eV_0/\hbar$  when there is a dc voltage  $V_0$  across it (due to fluxon motion); when that frequency is in the vicinity of the cavity resonance, a non-linear interaction with frequency locking may occur. Such a resonance leads to a current step at a fixed voltage in the I-V curve and to a series of steps at harmonics of this voltage. In the presence of a magnetic field an asymmetry relative to the junction is introduced due to the direction of the Lorentz force and the particle nature of the fluxon changes to an electromagnetic wave character; this leads to a series of steps known as Fiske Steps on the I-V curve.

The motion of fluxons is governed by a perturbed sine Gordon equation of the form

$$\phi_{tt} - \phi_{xx} + \alpha\phi_t - \beta\phi_{xxt} + \sin \phi = \eta \quad (4)$$

where  $\phi$  is the pair phase difference across the junction. The respective derivatives with respect to time  $t$  and position  $x$  are normalized to the junction plasma frequency and the Josephson penetration depth  $\lambda_J$ . The  $\alpha$  term corresponds to damping due to

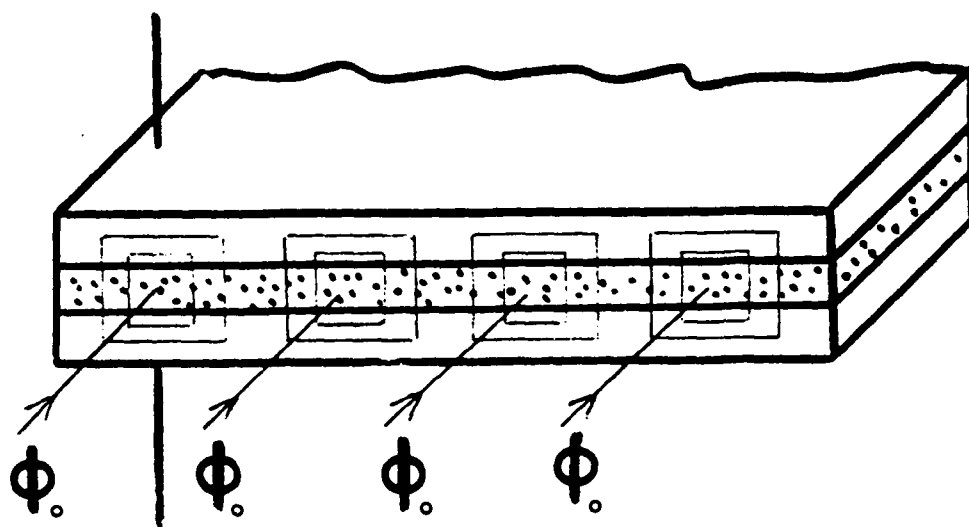


Figure 1: Fluxons in a long Josephson junction.



normal electrons and the  $\beta$  term due to surface losses. The right-hand side of equation 4 is the drive term, dc and/or ac bias. This equation represents well the behavior of a long junction and we will use it to analyze our results on the dynamics of fluxons and the effects of disorder. The terminal velocity of a fluxon is determined by a balance between the losses represented by  $\alpha$  and  $\beta$  and the energy input due to  $\eta$ . If we set  $\beta = 0$  (which can be questioned, as discussed later), the velocity is given approximately by

$$u = 1/\sqrt{1 + (4\alpha/\pi\eta)^2} \quad (5)$$

Because there are 2 basic frequencies, the Josephson oscillations and the cavity resonance, in the presence of non-linear effects, their interaction will lead to a variety of interesting phenomena. The very non-linear equation 4 leads to general solutions which are known as solitons, and for the case of long junctions these are fluxons.

### 1.3 Some Applications of Long Josephson Junctions

To put a better perspective on the research presented here, it is important to mention that there are already interesting applications of fluxon motion in long Josephson junctions. Some already important applications are:

- (i) microwave oscillator: the long junction can be dc biased at a current step and the reflections of fluxons at the edges will lead to fixed frequency radiation of microwaves. The steps occur at voltages given by

$$V_n = (\hbar/2e)n\omega_c \quad (6)$$

with  $\omega_c$  being  $\pi c/L$ ; here  $n$  is a positive integer,  $c$  the speed of the electromagnetic wave in the junction of length  $L$ .

This oscillator can be tuned by applying a constant bias current and varying an external magnetic field which leads to a varying voltage and hence to a change in the Josephson frequency. Thus the oscillation frequency can be tuned by both magnetic fields and bias currents. Power levels of  $1 \mu$  watt have been achieved<sup>8,9</sup> at frequencies tunable between 100 and 500 GHz.

- (ii) analog amplifiers: following a suggestion<sup>10</sup> that a long junction with current injection at many points in parallel is an almost complete analog of a FET, where the role of electric charge carriers is being played by fluxons, current gains have been achieved<sup>11,12</sup>. There are many interesting applications of this device in analog and digital electronics.
- (iii) digital information processing: the fluxon can be used as the basic bit of information<sup>13</sup>. It can be localized in a potential well created by perturbing electrodes across the junctions and it can be moved by applying currents and/or magnetic fields.
- (iv) non-linear studies: the highly non-linear equation 4 describing fluxon motion in a long junction can be a simple model for studying non-linear phenomena. The non-linear term  $\sin \phi$  in the equation makes it one of the most non-linear systems.

## 2. RESULTS

Studies have been made of the motion of fluxons in long Josephson junctions; such junctions can be considered as transmission lines or electro-magnetic cavities coupled non-linearly to fluxons. The results will now be described.

### 2.1 Telegraph Noise

The interaction of fluxon motion with cavity resonances of the junction leads to a series of current steps in the I-V curves of a long junction in a magnetic field. This is shown in Fig. 2 for a Nb-Al<sub>2</sub>O<sub>3</sub>-Nb junction, 130  $\mu\text{m}$  long and 6  $\mu\text{m}$  wide, whose current density is  $3 \times 10^3 \text{ A/cm}^2$ . Figure 3 shows a typical long junction as used in this research.

To investigate fluxon fluctuations, the junction was biased by a signal current in a control line over the junction. Current steps were observed in the I-V curves with a voltage periodicity of 47  $\mu$  volts. When biased on one of these steps, current fluctuations in the junction caused the voltage to fluctuate between this step and the adjacent one showing a characteristic telegraph type of noise. A typical telegraph noise voltage-time graph is shown in Figure 4. The power spectrum of the fluctuations between the 2 current steps can be represented by a Lorentzian distribution of the form

$$S(\omega) = 4 \bar{V} \Delta V \tau_0 / (1 + \omega^2 \tau_0^2) \quad (7)$$

where  $\Delta V$  is the voltage difference between 2 adjacent current steps and  $\tau_0$  is characteristic time describing transitions up and down between the 2 steps. Our measurements show that  $\tau_0$  can be in the range of  $10^{-6}$  second to  $10^{-2}$  second depending strongly on the bias conditions. The real-time voltage as shown in Fig. 4 forms a complete record of the trapping behavior for a fluxon in 2 different states. We have found that by making the width of the transition smaller, the lifetime  $\tau_0$  approaches the characteristic time for a fluxon to go across the junction,  $\sim 10^{-10}$  seconds.

We have created a situation, with the proper bias, where a fluxon can hop between 2 states, its amplitude being determined by the

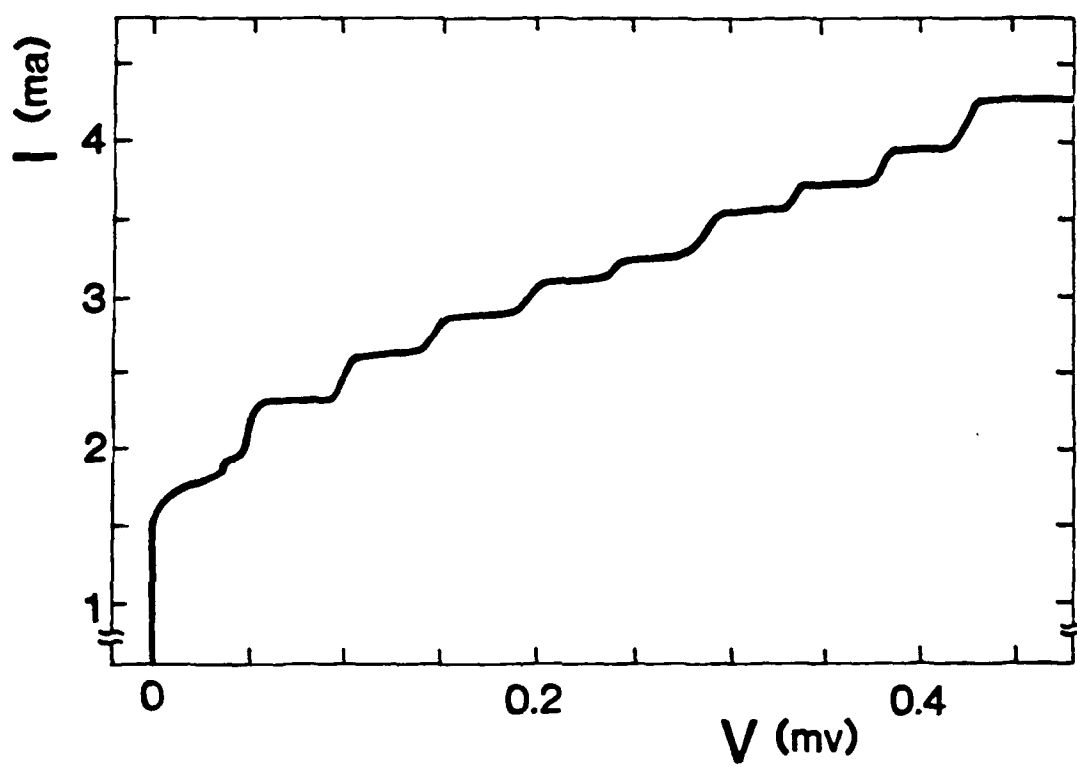
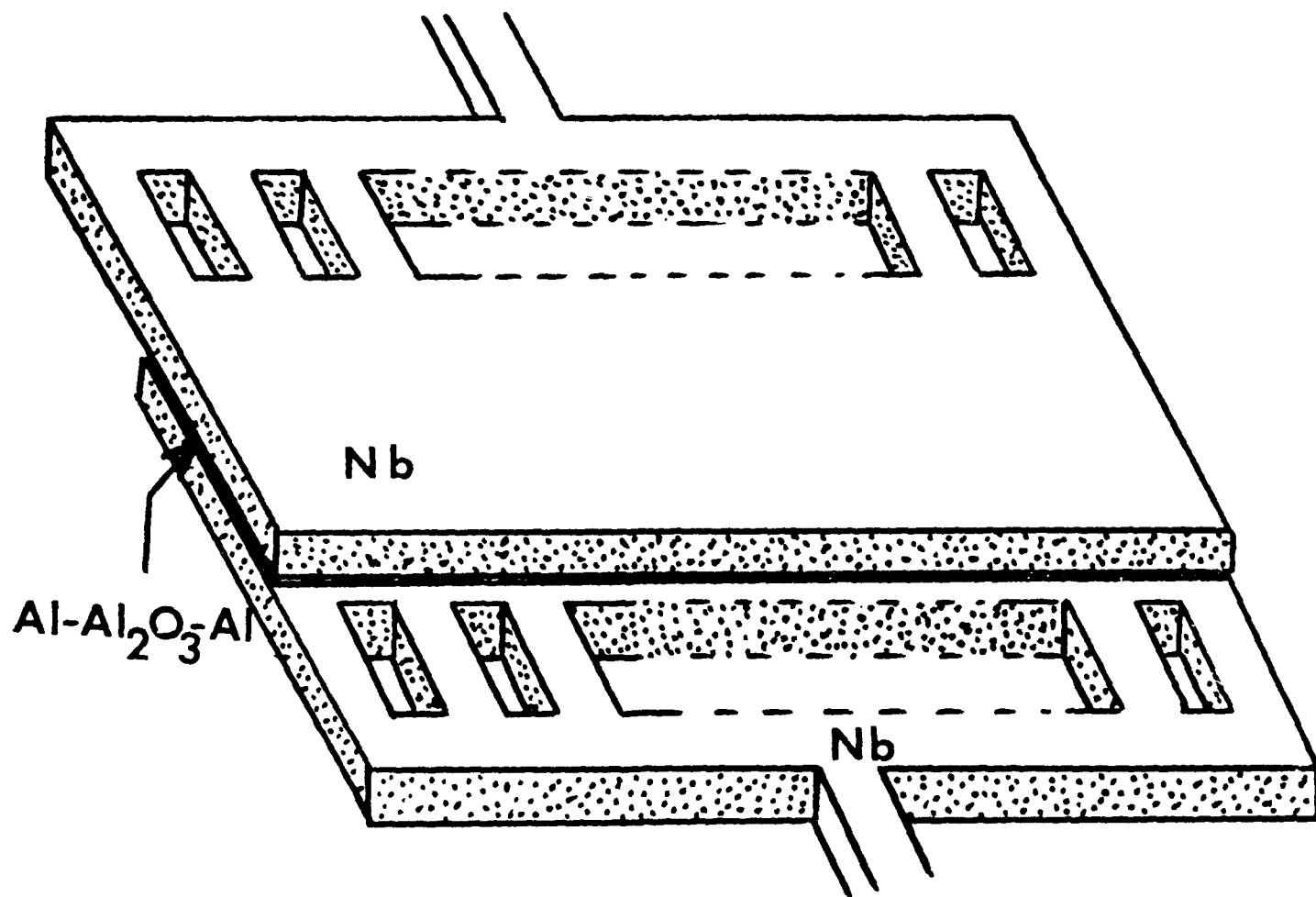


Figure 2: Current steps on I-V curve of a long junction in a magnetic field.



## LONG JUNCTION

Figure 3: Long Josephson junction with 8 current-injection fingers.

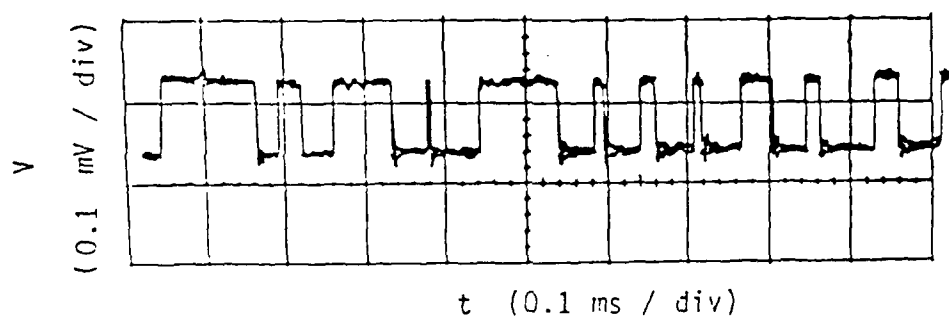


Figure 4: Telegraph noise at a current step.

junction geometry, i.e., its length  $L$ . The drive mechanism responsible for this is of thermal origin, at least between 4 K and 1.5 K.

Telegraph noise has been studied as well in other systems ranging from amorphous silicon with individual electron traps, to silicon inversion layers with interface traps, to electron traps in small Josephson junctions, and to SQUIDS; our results<sup>18</sup> deal with fluxon traps at current steps where we have investigated the drive mechanisms. The long junctions provide an excellent model for studying the decay of metastable states (between 2 current steps) classically and quantum mechanically as well as for investigating fluxon dynamics. Our results show how the fluxon lifetime in a given state is characterized by the current uncertainty for switching to a higher level and by thermal effects.

## 2.2 Effect of Ultrasonics on Resonant Modes of Junction

This was an exploratory experiment performed in order to investigate the effects of ultrasonics on fluxon motion and fluxon pinning. Since fluxon pinning in junctions exists, as evidenced in the previous experiments, we were interested in using ultrasonics to reduce this. An ultrasonic transducer was attached to a long junction, with electrostatic shielding between the two. The transducer operated at 1 MHz producing longitudinal excitations. As the ultrasonic level increased, the current step structure decreased and eventually disappeared. Although we were hoping to modulate the bias current by the ultrasonics and thus to modulate the force on the fluxons and their resonance condition, this experiment was not conclusive. It should be repeated with better coupling to the junction. The ability to modulate the pinning defects or the critical current with ultrasonics would provide interesting phenomena for basic and applied studies.

## 2.3 Effects of Geometry on Gain of Long Junction

In order to have a large and uniform current flowing through the long junction, current was injected by means of 8 fingers as shown in Figure 3. Such a measure is necessary as otherwise the current would be forced, by self-induced fields, to flow mainly at the edges. The current gain of such a junction can then be written as

$$G = \frac{\Delta I_b}{\Delta I_c} \quad (8)$$

which is equivalent to  $r_m/R_D + R_L$ . Here  $R_L$  is the external load,  $R_D$  is the dynamic resistance of the junction, and  $r_m$  its transresistance given by

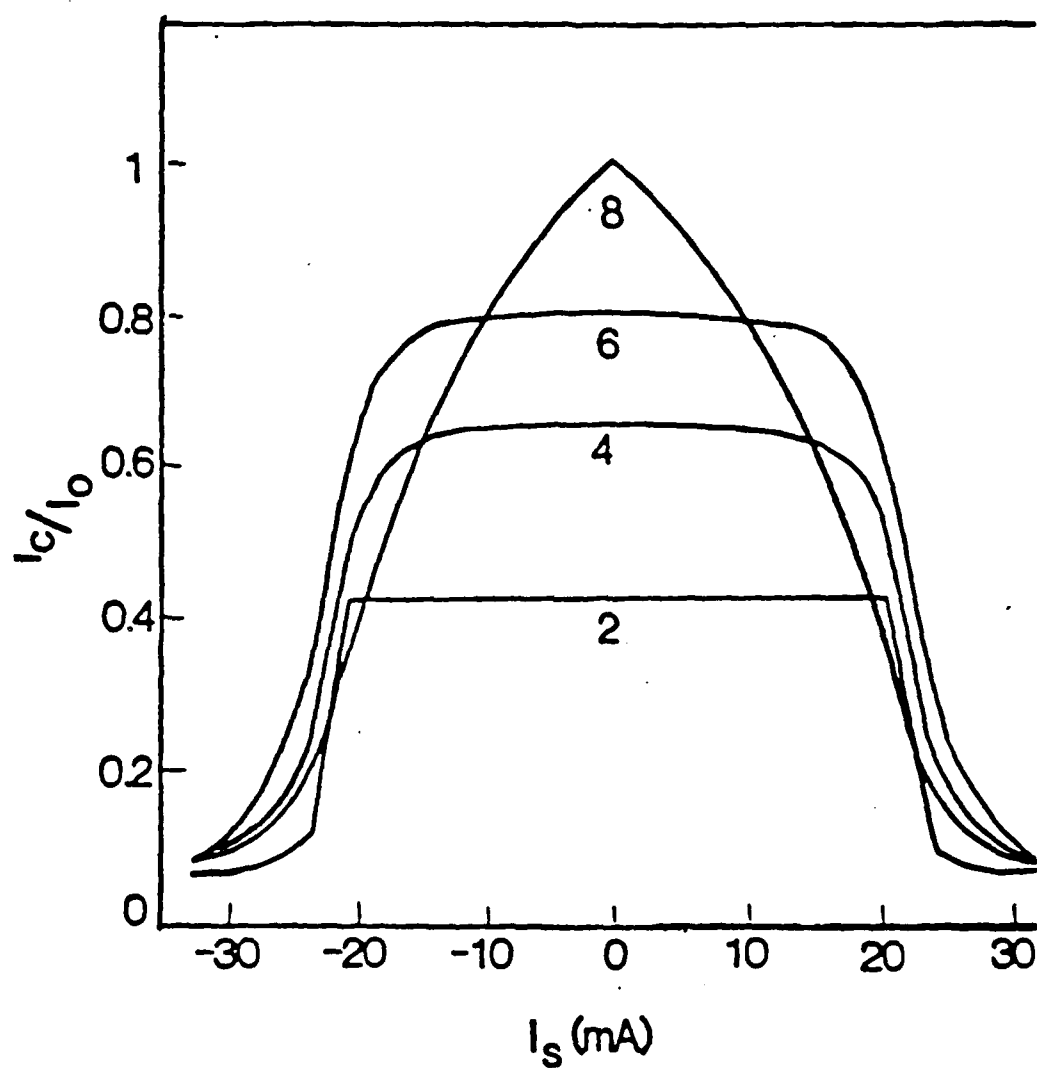
$$r_m = \mu_o d \bar{c}/w_s \quad (9)$$

The velocity of the electromagnetic wave in the junction is  $\bar{c}$ ,  $d$  is the penetration depth of the magnetic field,  $\mu_o$  the permeability, and  $w_s$  the width of the control line.

A reduction of  $w_s$  will cause an increase in gain. Indeed we have observed that by reducing the width of the control line from 65  $\mu\text{m}$  to 19  $\mu\text{m}$ , the gain increased by a factor of 3.

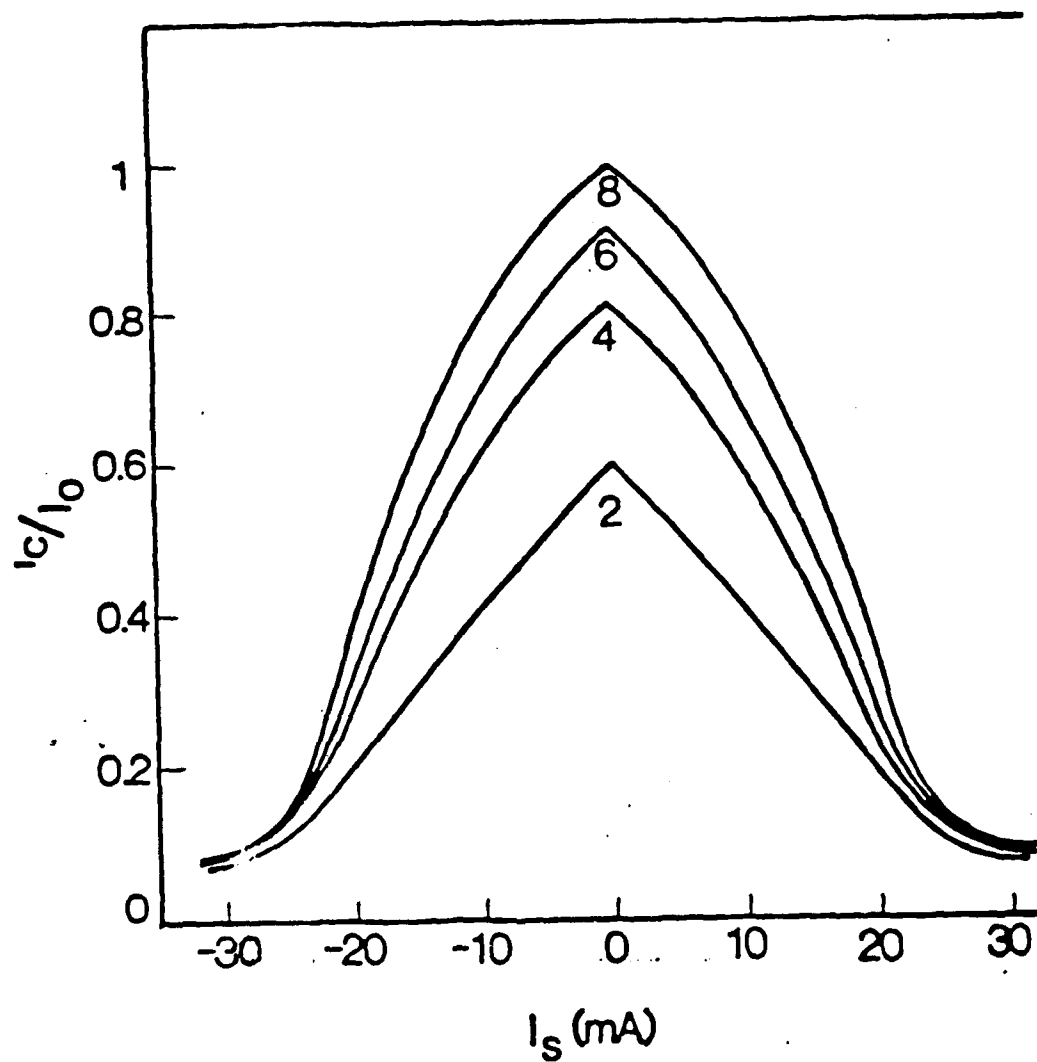
Also, by increasing the number of fingers injecting current into the junction, the current gain increased. The location of the fingers was very important to the gain. Figure 5 shows the gain characteristics of the junction when fingers are removed progressively from the ends. Figure 6 shows the gain characteristics when the fingers are removed from the center. The choice as to where the current injecting fingers should be located depends on the application; in one case the dynamic range is large, while in the other case sharp switching can occur. The





NEAR CENTER

Figure 5: Gain characteristics of a junction when current injection fingers are removed from the ends.



NEAR EDGES

Figure 6: Gain characteristics of a junction when current injection fingers are removed from the center.

results also show that current gain is raised by increasing the number of fingers.

#### 2.4 Metastable States in Junction

When an external magnetic field is applied to a long junction, the I-V curves show current steps, known as Fiske steps. They are due to the resonant excitation of cavity modes in the junction. The current steps can be considered as metastable states when biased near a transition to the next state; we have studied fluxon fluctuations at such states. Actually noise-activated escape from metastable states was treated a long time ago by Kramers and has since been applied to many systems. We have extended these ideas to fluxons in long junctions where we investigated the origin and the dynamics of fluxon fluctuations. Figure 7 shows the metastable region on a portion of the I-V curve for the junction. There are applications (ex. fluxon oscillator) where such a junction is biased at a current step; fluctuations<sup>16</sup> in current or limited microwave radiation linewidth are of practical interest there. We have investigated the probability of fluxon escape from its equilibrium level to a higher level and how it depends on defects, bias and dissipation. The studies deal with fundamental questions of thermal decay rates, quantum decay rates, energy levels, and possible non-equilibrium situations.

#### 2.5 Temperature Dependence of Fluctuations

We have extended the above fluctuation studies to the temperature range of 4.2K to 1.5K. A special cryostat was constructed for these studies with electrical and magnetic shielding for the devices. Figure 8 shows how the transition width  $\Delta I$  at a current step changes with temperature. The lower the temperature the narrower is this current width, thus leading to a higher transition probability per second for switching to the next state when a junction is biased within  $\Delta I$ . A plot of the maximum probability as a function of temperature shows a logarithmic behavior as presented in Fig. 9. Such temperature dependence is

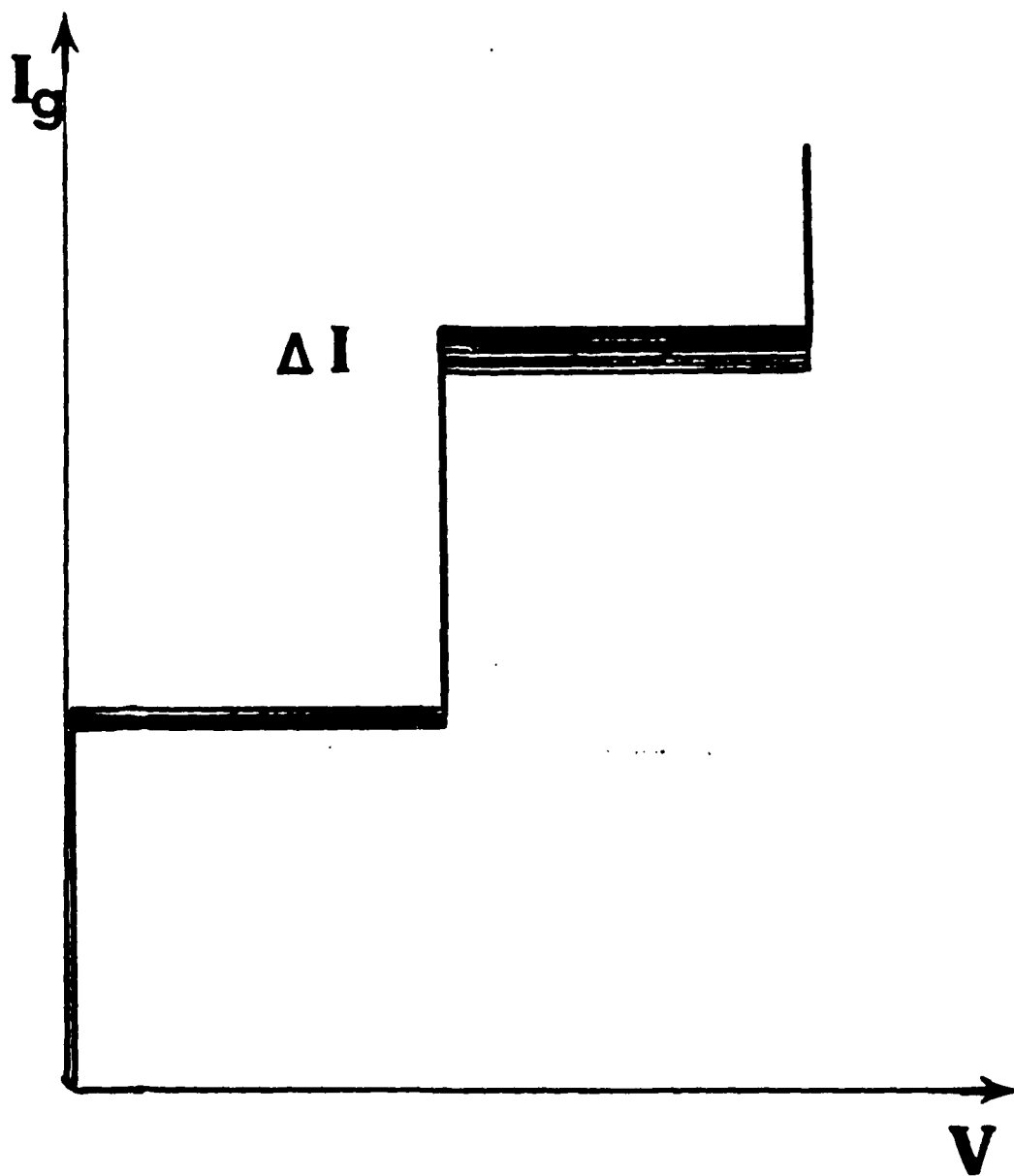


Figure 7: Metastable regions at current step of a long junction.

characteristic of thermally activated processes responsible for the escape of fluxons from one state to the next one<sup>17</sup>. Because the critical current of the junction varies with temperature, its dependence had to be taken into account in our fit of the data.

By biasing at a current step  $I_n$  and watching the junction fluctuate to the  $I_{n+1}$  step, we have created 2 potential wells between which the fluxon can jump or tunnel through, depending on the temperature. Analysis of such fluctuations shows:

- (a) the power spectrum of the noise is a Lorentzian distribution characteristic of a 2-level system.
- (b) the mean lifetime for the 2 levels increases with the current width  $\Delta I$  over which a step is bistable.
- (c) in the temperature range of 4K - 1.5K the current width  $\Delta I$  varies with temperature as  $T^{2/3}$ .
- (d) the lifetime  $\tau$  of a fluxon remaining in the lower state has an exponential temperature dependence of the form

$$\frac{1}{\tau} = \frac{\omega_1^2}{2\pi\eta} e^{-\frac{\Delta u}{kT}} \quad (10)$$

where  $\omega_1$  is the attempt probability and  $\eta$  the flux flow viscosity.

The results show that we have the classical problem of a particle, the fluxon here, with thermal energy  $kT$  trying to escape over a barrier  $\Delta U$  which is larger than  $kT$ . This is the thermal activation regime. Such a model has been applied to regular small junctions where fluctuations occur from the zero voltage state to the voltage state (the energy gap); in our case we have fixed, by the bias conditions, the barrier of the potential wells and the fluctuations occur between adjacent voltages states closely spaced, making the interpretation simpler. The fluxon lifetime in

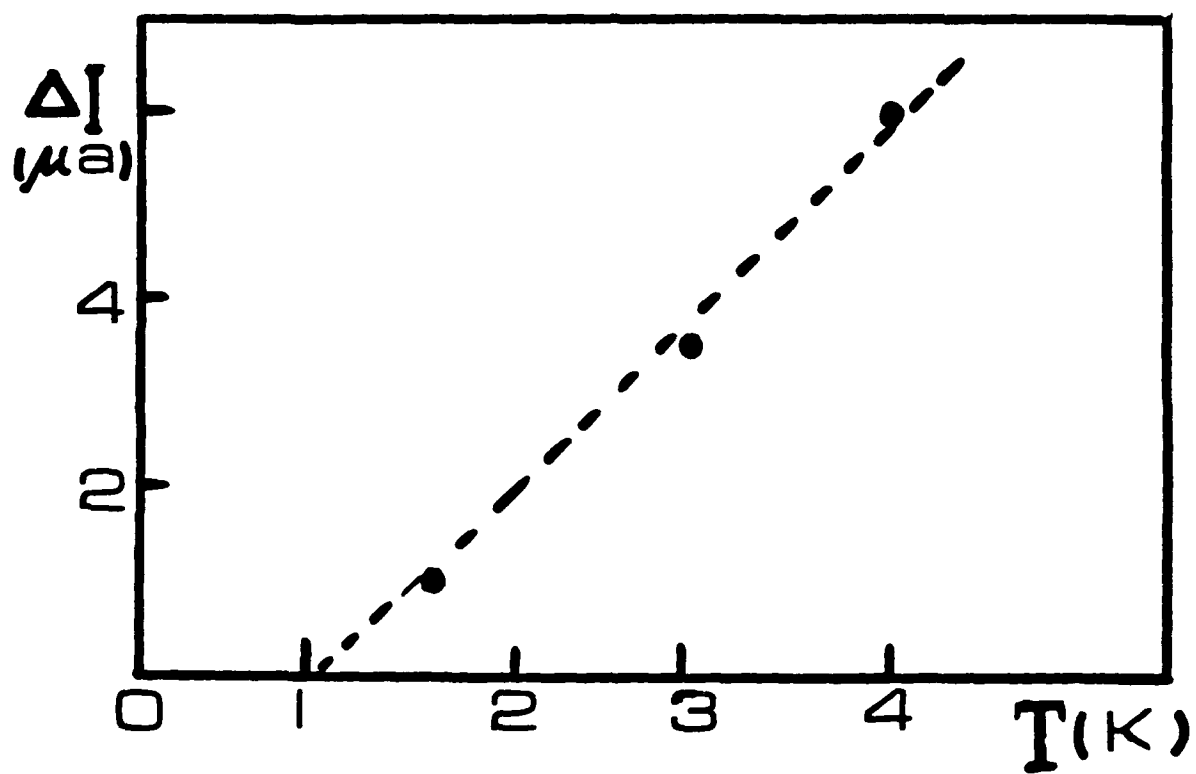


Figure 8: Temperature dependence of step width.

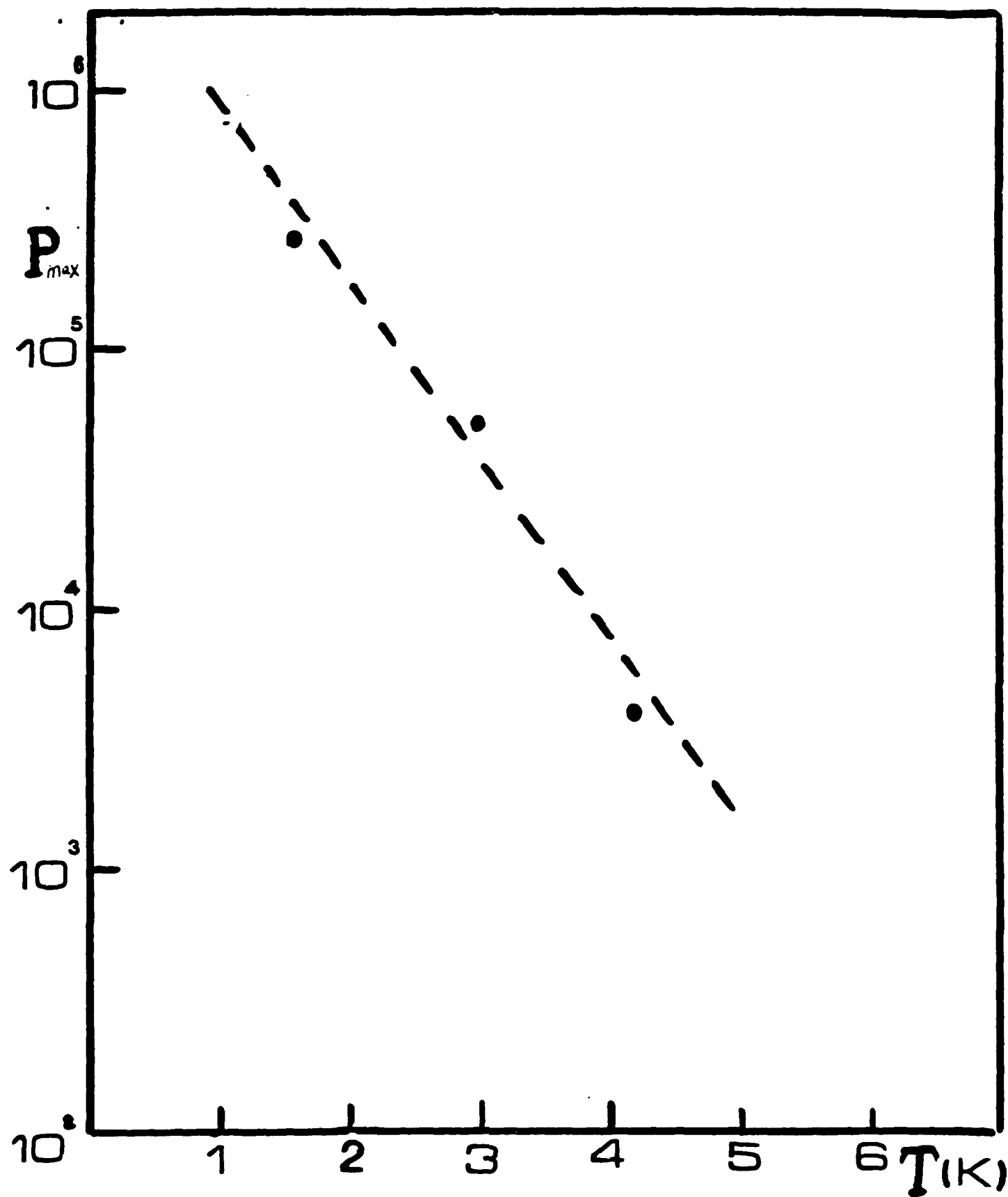


Figure 9: Temperature dependence of maximum probability.

a potential well strongly depends on the barrier dissipation. In small junctions, current steps are usually induced by external microwaves; in our case the microwaves are internal to our junctions and they also induce steps.

## 2.6 Junction Dynamics and Chaotic Behavior

While measuring the I-V curves of our long junctions in a magnetic field we observed a large amount of noise on some parts of the curve, as shown in Figure 10. Although this is a trace from a X - Y chart recorder, it is clear that there is quite a bit of noise, beyond the bandwidth of the recorder. Figure 10 is typical for junctions with  $\beta_c > 1$ . According to equation 4, we have a non-linear system with some dissipation and it is driven by the bias current; the system can exhibit instabilities leading to non-periodic or chaotic time evolutions which arise from the internal dynamics of the system. We have here one example (a very good one) of a non-linear system exhibiting a complicated but interesting behavior. From a general definition of chaos, which refers to a noisy time-dependent motion, it is just as shown in Figure 10. Many questions arise, one of them being about the instabilities that precede the chaos and their nature. Although some systems do not show instabilities, many do show well-defined instabilities. The various sequences of instabilities leading to chaos can be identified as:

- (a) period doubling (the basic period of oscillation doubles repeatedly)
- (b) quasi-periodicity (several interacting oscillations at incommensurate frequencies)
- (c) intermittency (periodic and non-periodic phases alternate in time).

To understand how our results are related to theoretical predictions, we have modeled our junction on the computer using equation 4 and suitable boundary conditions:



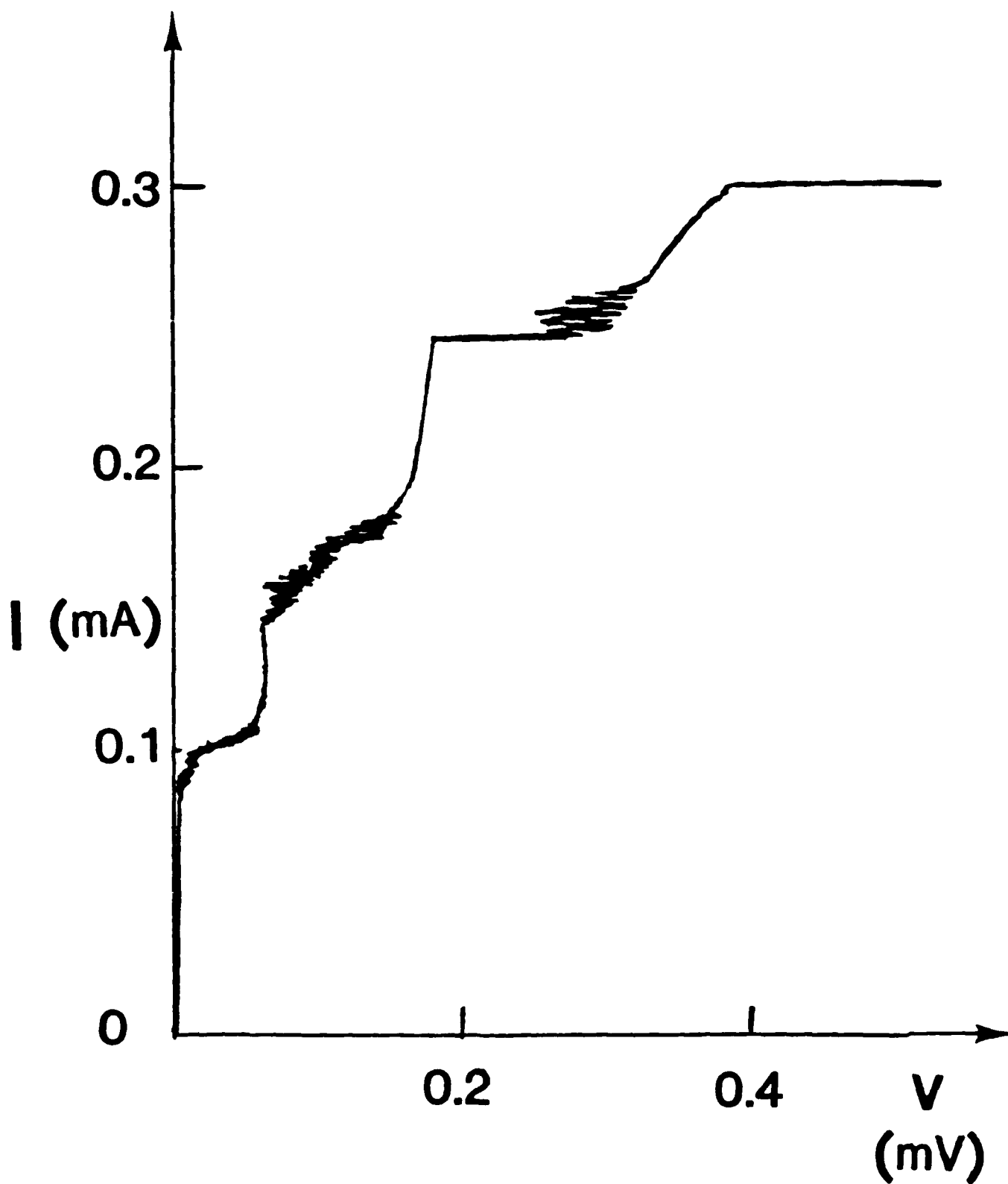


Figure 10: Excessive noise on a current step.

$$\phi_x(0,t) - \phi_x(L,t) = \eta \quad (10)$$

where  $\eta$  is the normalized external magnetic field applied perpendicular to the junction long dimension. This is a complicated equation to solve as it takes much computer time to get numerical solutions. We have used a finite difference method to solve it. In a regular long Josephson junction of the overlap type, the bias current is concentrated within a distance  $\lambda_J$  from the edge of the junction, i.e., the current distribution is not uniform. To overcome this deficiency, we use 8 injection fingers for bringing the current at the top electrode and 8 fingers to take out the current. This type of current injection makes the current distribution in the junction more uniform, thus providing a more uniform Lorentz force. However, the presence of the equally spaced fingers must perturb the fluxon motion, since each pair of fingers can be considered as a small potential well (this is the principle of the fluxon ring storage device). Static measurements of the gain characteristics show that the position of the fingers has a tremendous influence (we presented this in Section 2.4). To study this problem we looked at the dynamic effects using equation 4 in a computer simulation. The results show that the dynamics is essentially independent of how the current is distributed in the junction due to the averaging of the Lorentz force by the travelling fluxons. This is shown in Figure 11, where the behavior of one fluxon is essentially unaffected, except for the small ripples; the same applies for the reflected fluxon. The graph shows a fluxon for various times at different positions in a junction whose length is  $L = 6\lambda_J$  and with no external field. This is in good agreement with recent calculations and laser scanning experiments showing the junction current distribution. The small oscillations are probably due to microwave excitations in the barrier as the fluxon travels. Figure 12 shows the calculated I-V curve, corresponding to our junction with self-generated current steps. This graph also shows that some regions have periodic solutions, while others are chaotic.

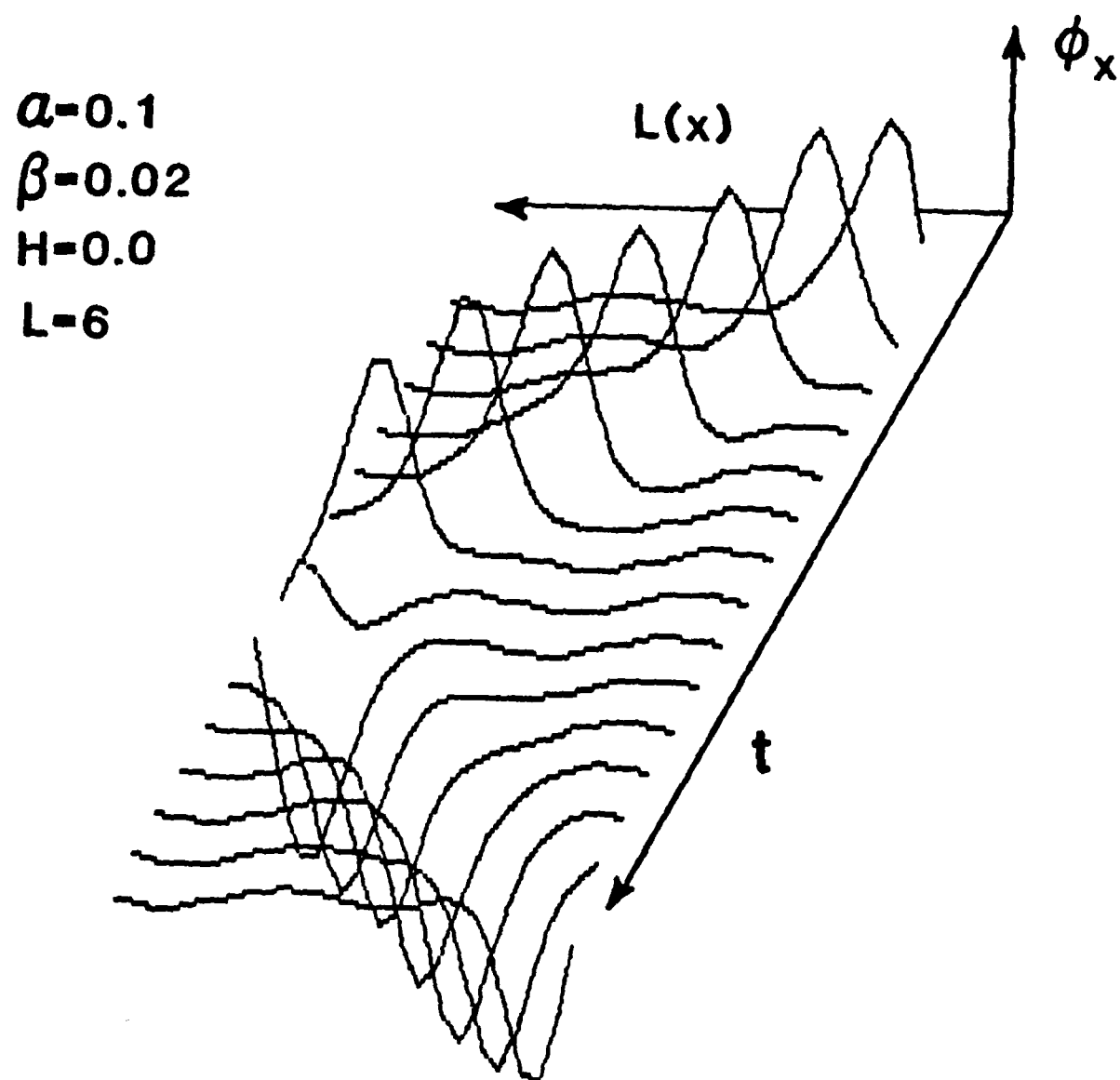


Figure 11: Dynamics of a fluxon in a long junction unaffected by the bias current distribution.

The behavior depends on the order of the steps. We are probably the first to show the fluxon periodic and chaotic behavior in a long junction. It is clear that in some regions the device should not be biased as the behavior will be unstable.

To distinguish the behavior between periodic and chaotic, Figure 13 shows the periodic behavior while Figure 14 shows the chaotic behavior. This can be demonstrated in the power spectrum and it is shown in Figure 15 for periodic solutions and Figure 16 for chaotic ones. It is interesting to note that as the bias point moves by a small amount, from  $J = 0.42$  to  $J = 0.44$ , the behavior goes from periodic motion to chaotic behavior.

Our results also show, for the first time, that on a given current step there can be chaotic and periodic behavior, depending on where the bias is set. Intermittent chaos between Fiske steps was the only result reported<sup>19</sup> so far. We show regions of chaotic behavior for a long junction with a constant external magnetic field and no external ac driving term. It occurs even within Fiske steps where the junction could be biased for some application.

From this study, an interesting question arises as to how chaotic behavior is approached. Figure 17 shows the power spectrum at a bias of  $J = 0.4306$ . This spectrum (and others) indicate that chaos is approached by a period doubling bifurcation; in this sequence of instabilities, Figure 17 shows the period 8 bifurcation with very noisy behavior.

We conclude that chaotic behavior is observed for long junctions with  $\beta_c > 1$  by a period doubling bifurcation sequence.

## 2.7 Period-Doubling in a Perturbed Sine-Gordon System

It has been found that low dimensional dissipative non-linear systems show chaotic dynamics which can be approached through

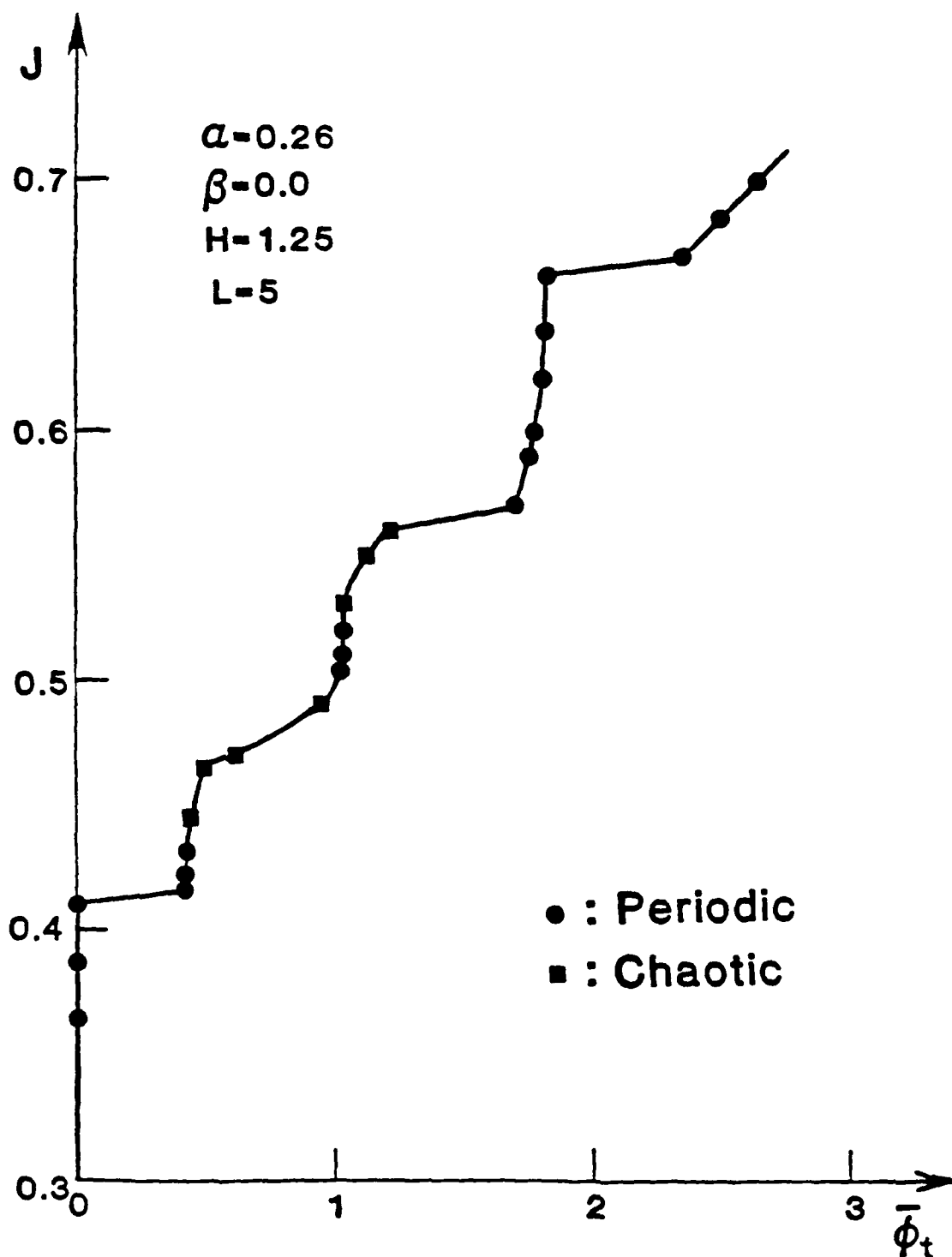


Figure 12: Computed I.V. curve for our junction with regions of instability.

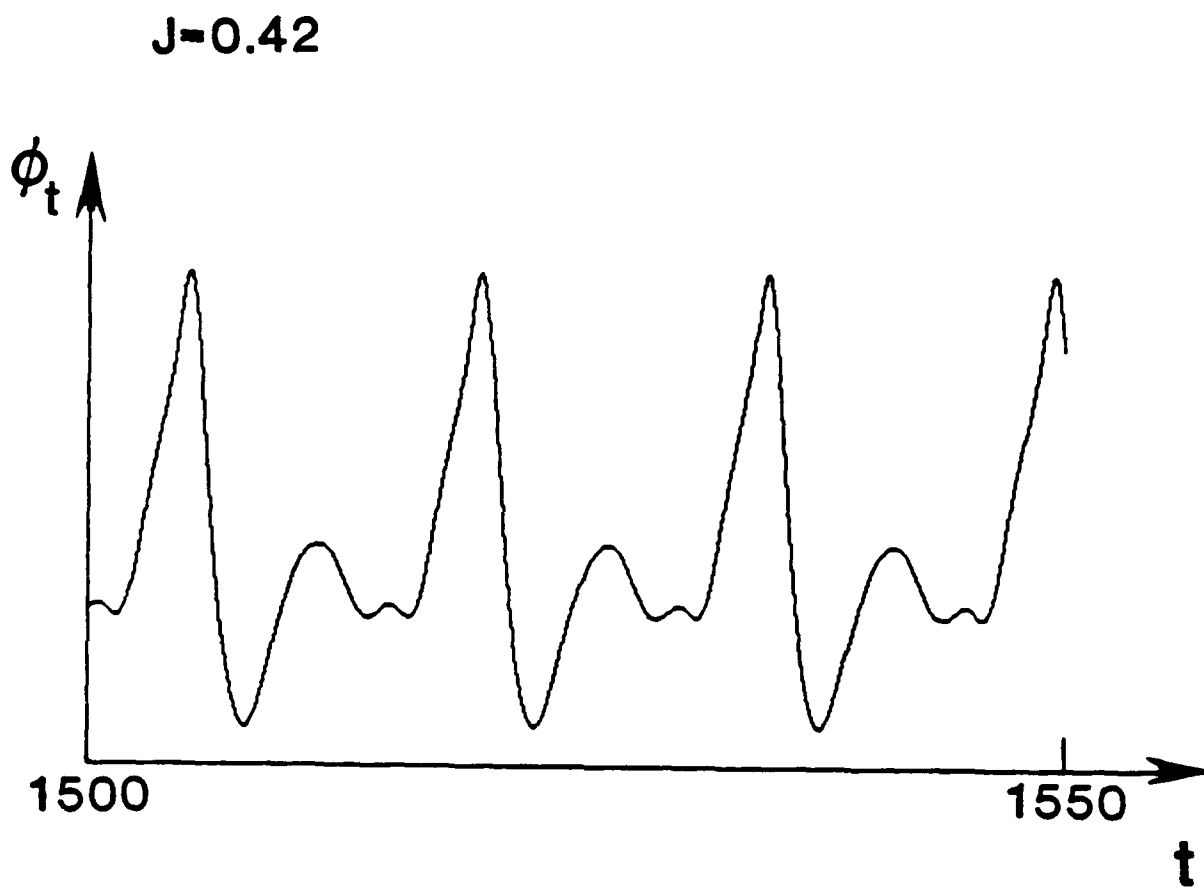


Figure 13: Periodic behavior of  $\phi_t$  for junction biased at  $J = 0.42$ .

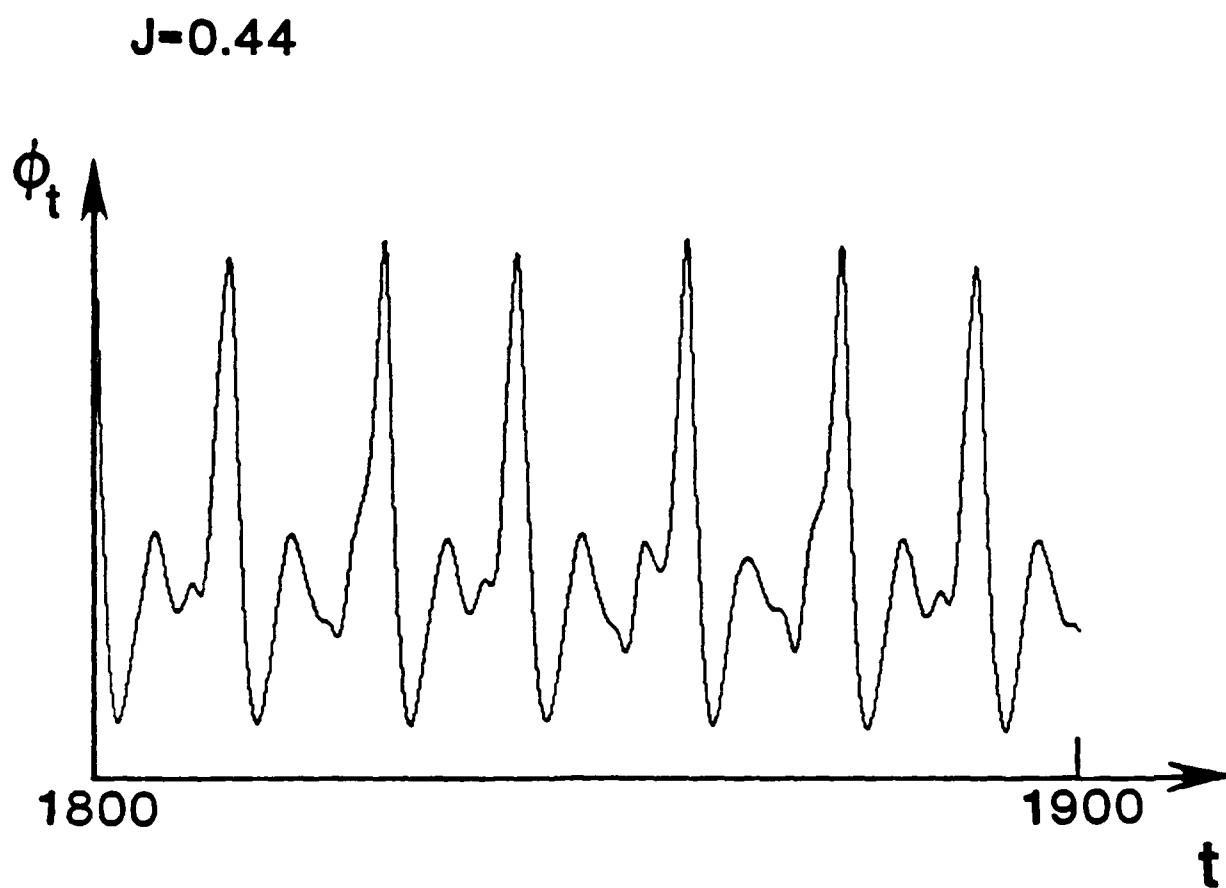


Figure 14: Chaotic behavior for junction biased at  $J = 0.44$ .

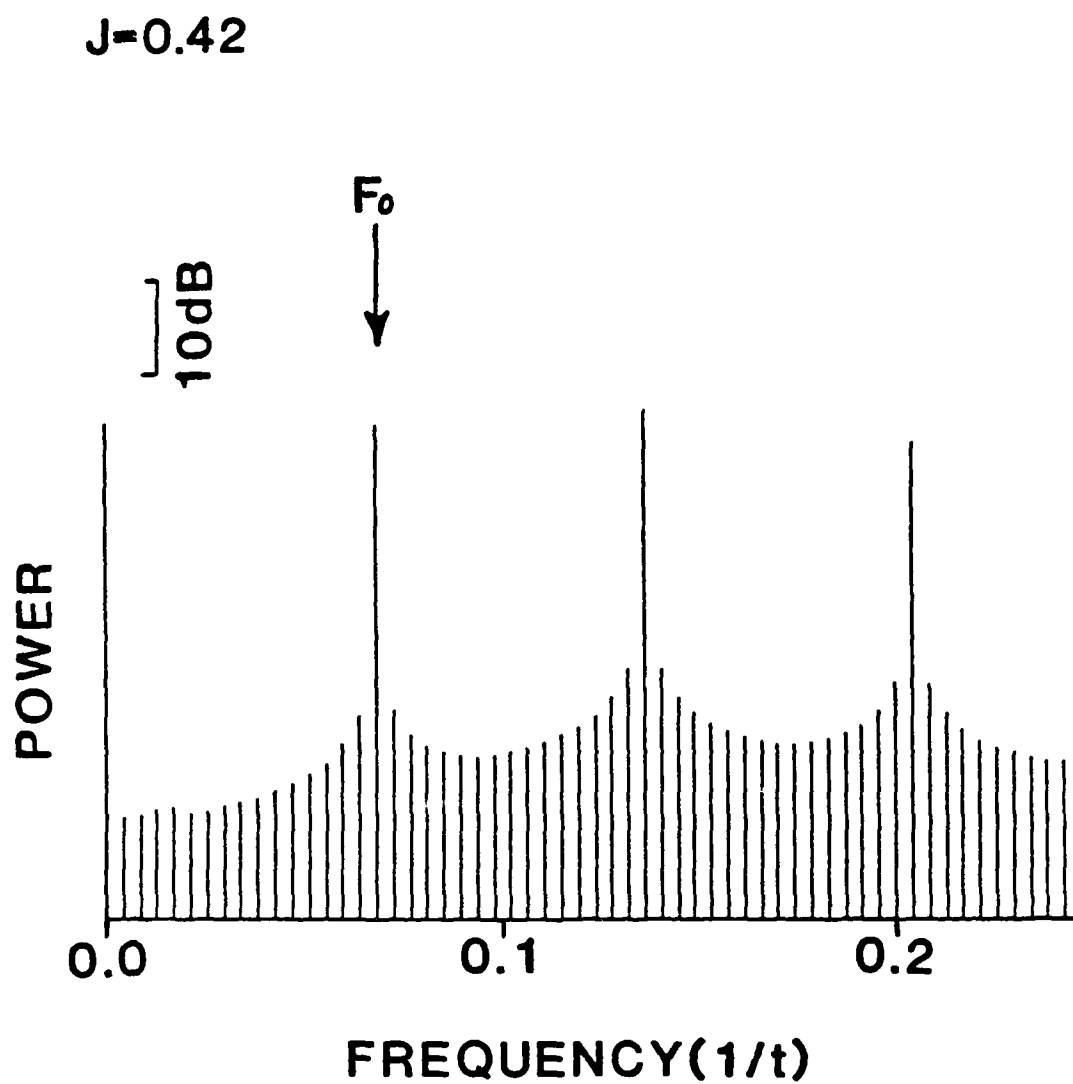


Figure 15: Power spectrum for periodic motion.



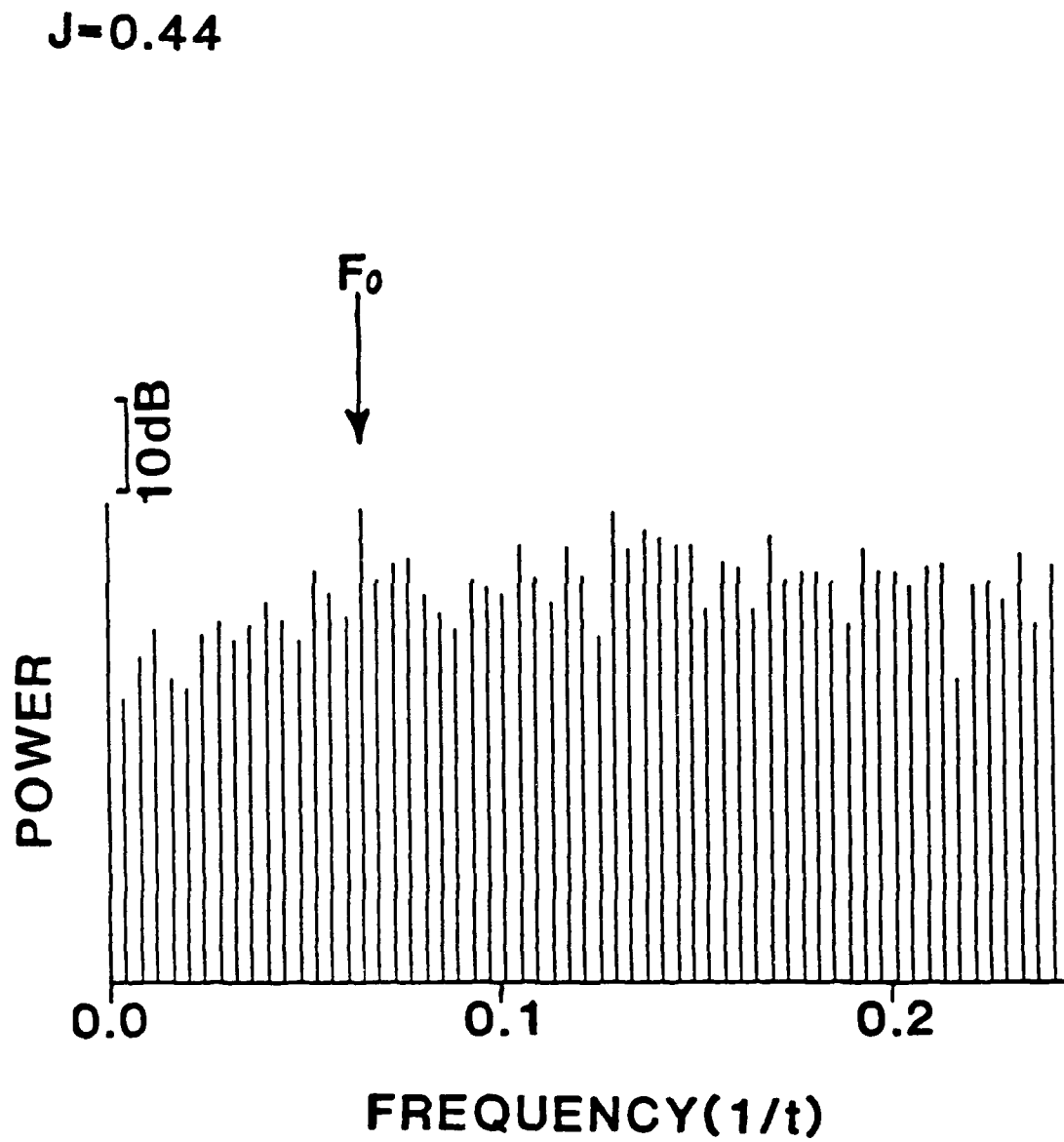


Figure 16: Power spectrum for chaotic motion.

$J=0.4306$

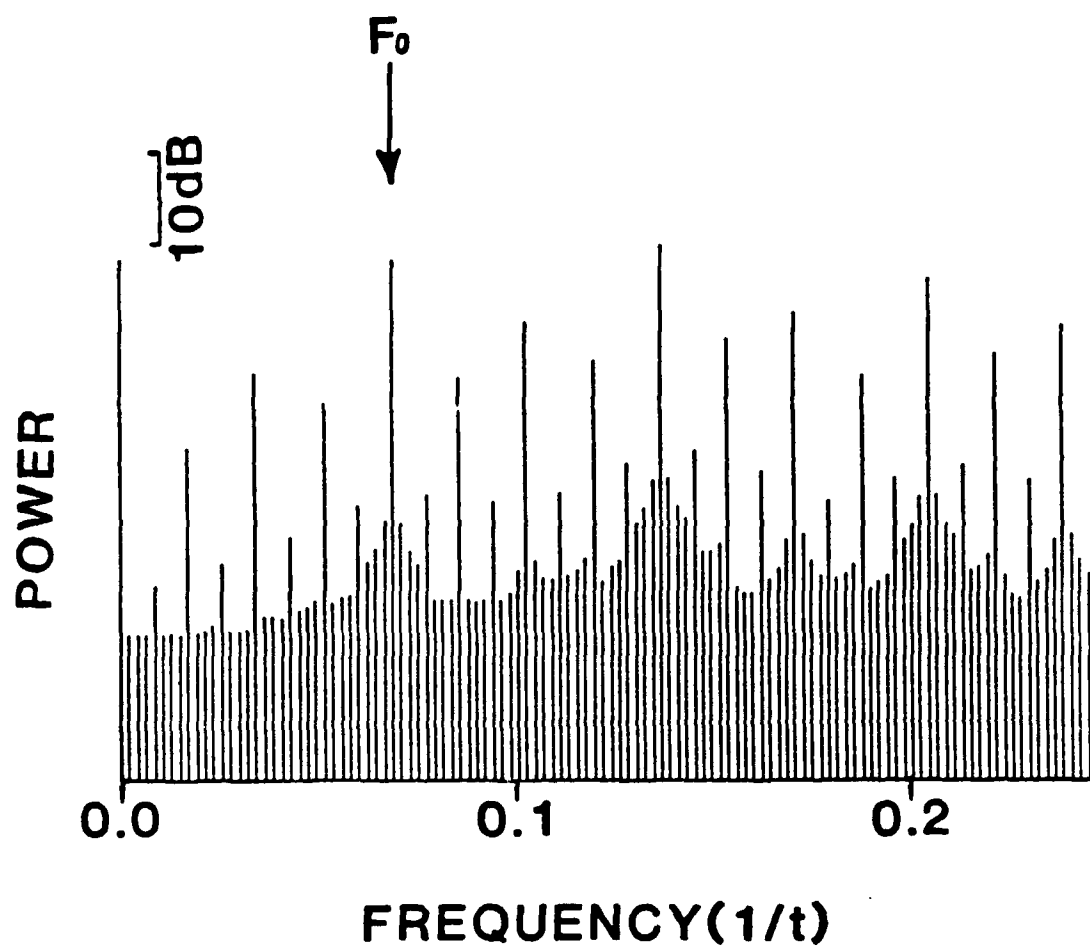


Figure 17: Period-8 bifurcation on part of I-V curve from computer modeling on first substep.

universal behaviors. The perturbed sine-Gordon equation is one such system. Although it describes many situations, we have used it in this research to describe the motion of fluxons in long Josephson junctions (equation 4). Temporal and spatial chaos can be found under the influence of spatially uniform or non-uniform driving forces. We have found that the route of period-doubling bifurcation sequence to chaos exists in the perturbed sine-Gordon system without r.f. driving force; the chaotic regime is within the soliton dynamic state. To the best of our knowledge, this is the first time that in a sine-Gordon system the route of period-doubling bifurcation to chaos is found. This result clearly demonstrates that the chaotic motion described here is really low dimensional. The agreement of our experimental results with computer modeling is excellent.

When the drive  $\eta$  is increased from zero, the system is first at the zero voltage state until  $\eta = 0.417$  is reached. At this point the system switches into one of the first Fiske steps with very good periodic motion. We find that for a narrow range of  $\eta$  (0.417 to 0.43) solutions are stable and the fundamental frequency is equal to the Josephson frequency. When  $\eta$  is larger than 0.43 the system starts to bifurcate, the fundamental period is doubled. As  $\eta$  is further increased, the system passes through period 4, and period 8 bifurcation. Beyond that, the system is in a reverse set of bifurcations in the chaotic regime. Figure 18 shows the power spectrum at the situation with  $\eta = 0.436$  where the  $f_0/8$  peaks have bands of broad noise indicating that the system is in the noisy period-8 region. The broadband noise continues until the system enters the fully developed chaotic region at  $\eta = 0.44$ .

At 0.453 the system jumps into another periodic Fiske Step 1 mode, with a soliton moving in one direction and reflecting as a plasma wave. This is the main Fiske 1 mode, the previous one being a sub-Fiske 1 mode. The basic frequency at  $\eta = 0.453$  is half the Josephson frequency which indicates that the system has already bifurcated. Further increase in  $\eta$  leads to another chaotic regime

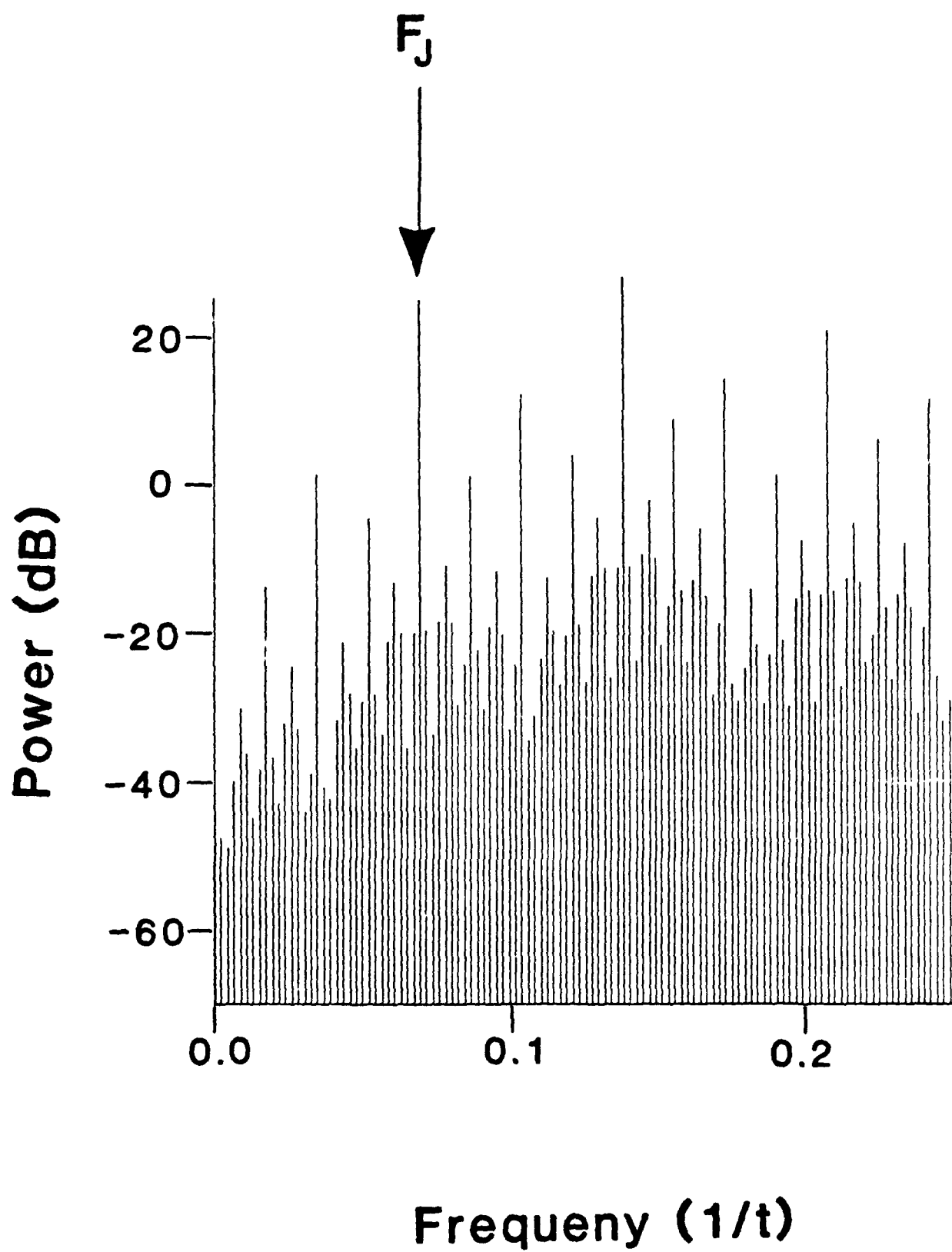


Figure 18: Period-8 bifurcation at first substep.

through a period-doubling bifurcation sequence. Fig. 19(a) shows period-16 bifurcation at  $\eta = 0.457$ , followed by chaotic behavior at 0.46, Fig. 19(b). This is followed by a period-3 window shown in Fig. 20. After this window is passed, the system goes in a chaotic regime again.

Another interesting feature of our simulations is the  $\eta$  vs.  $\langle \phi_c \rangle$  curve which corresponds to our experimental I-V curve; this is shown in Fig. 21 where the results were computed for the Fiske step 1 region. As soon as the system enters the chaotic regime in both Fiske steps, the dynamic resistance goes to a negative value for a small range of  $\eta$ .

Usually, period-doubling bifurcations are very difficult to detect directly in real Josephson junctions due to the very high frequencies and the very low power levels associated with the Josephson oscillations. In our experimental results for an overlap Nb-Al<sub>2</sub>O<sub>3</sub>-Nb long tunnel junction, we do observe a negative resistance region at the end of one Fiske step. This is shown in Fig. 22. As soon as the system enters the negative resistance range the noise<sup>20</sup> becomes very large. Actually the occurrence of a negative dynamical resistance in Fiske steps accompanied by a large amount of noise may serve as an indicator of a chaotic regime approached from period-doubling bifurcations. Our computer simulations do support and clarify our experimental results on long junctions.

## 2.8 Amplification in a Period-Doubling Bifurcating Dynamic System

There is a wide variety of non-linear systems showing period-doubling bifurcations with instabilities leading to chaotic behavior<sup>21</sup>. We have shown here that this is true for long Josephson junctions. The presence of broadband noise just before the bifurcation starts suggests that in that region, near the onset of a period doubling, the system not only amplifies noise

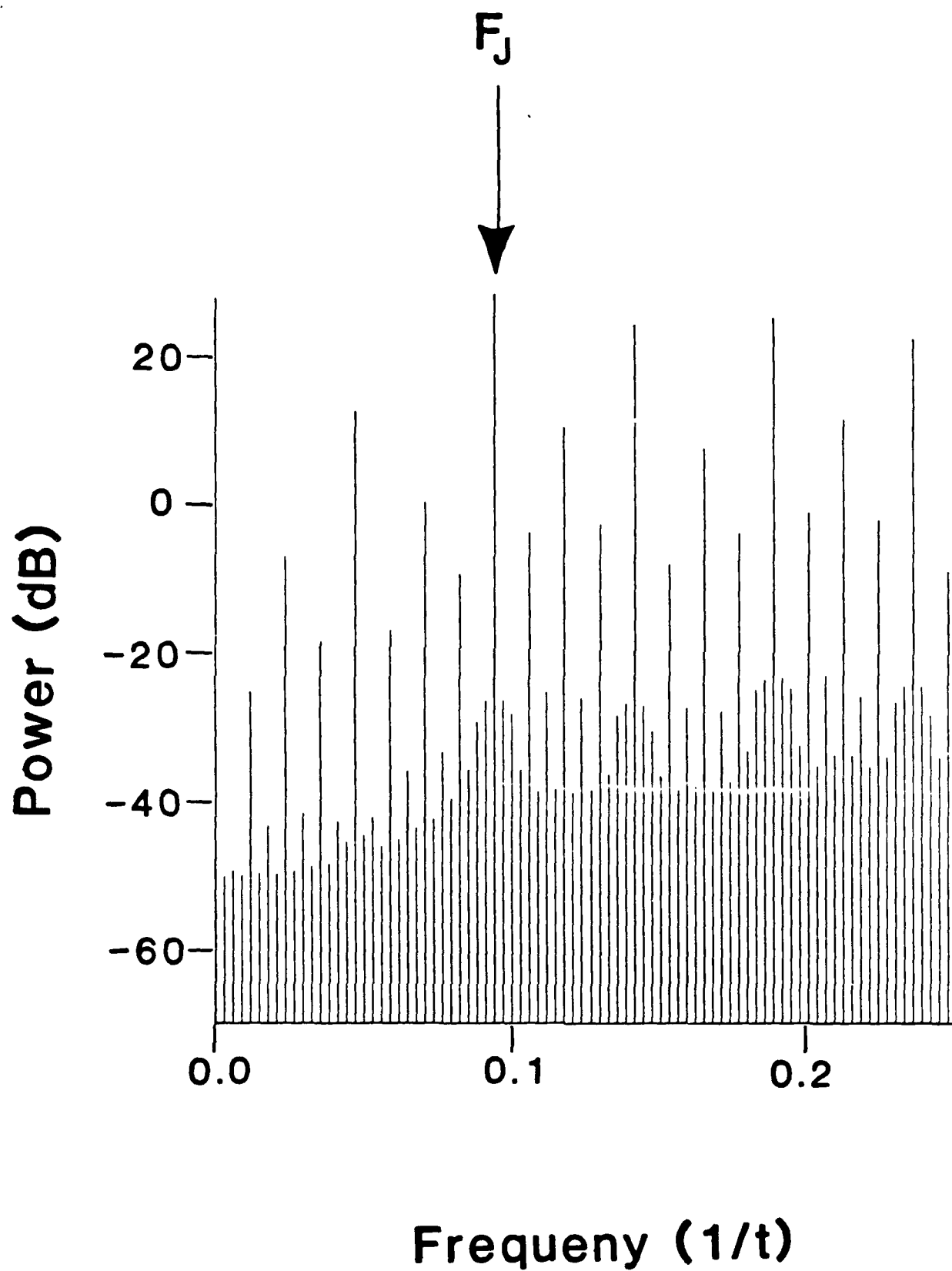
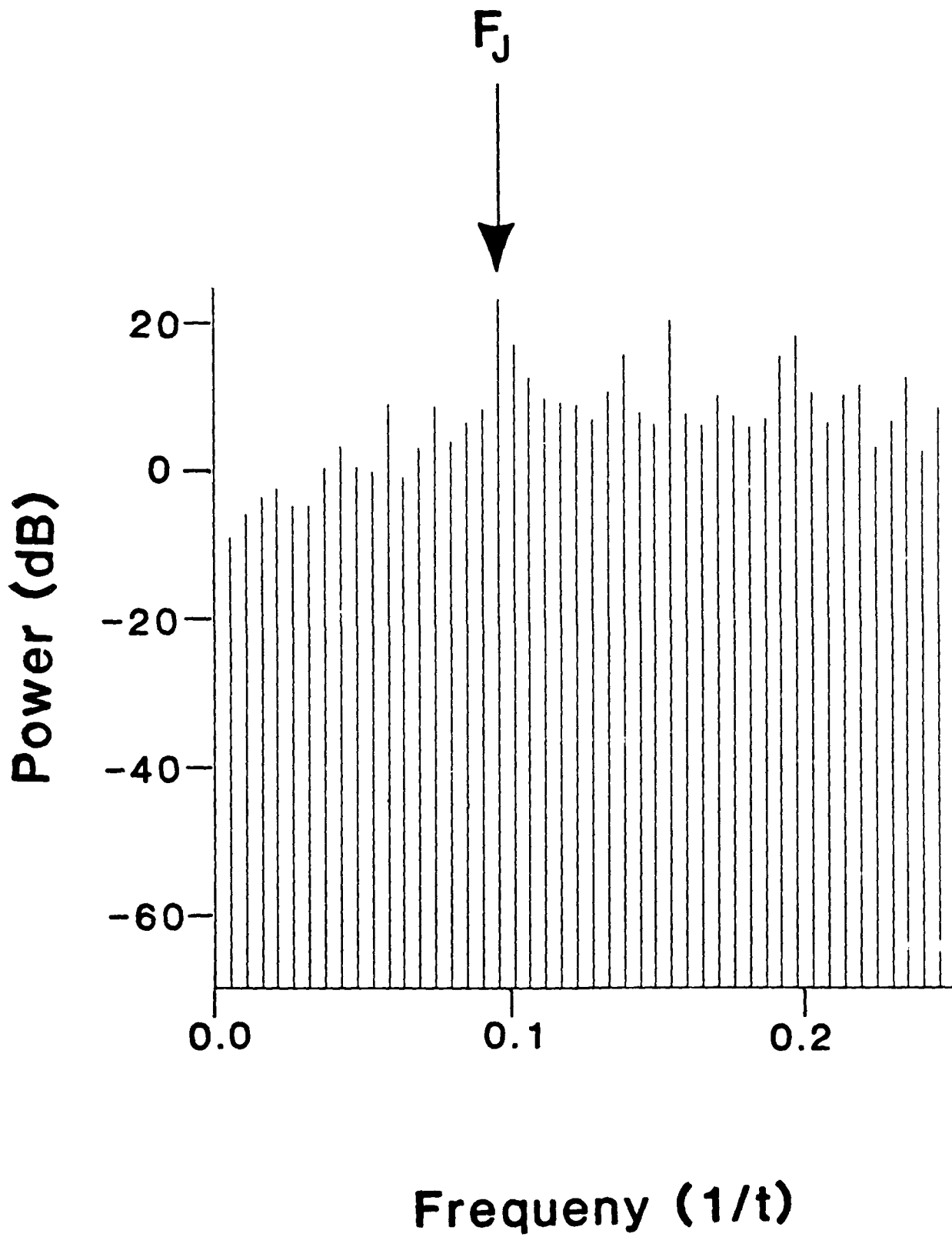


Figure 19: (a) Period-16 bifurcation on main Fiske step 1.



(b) Chaotic behavior at  $\eta = 0.46$  on main Fiske step.

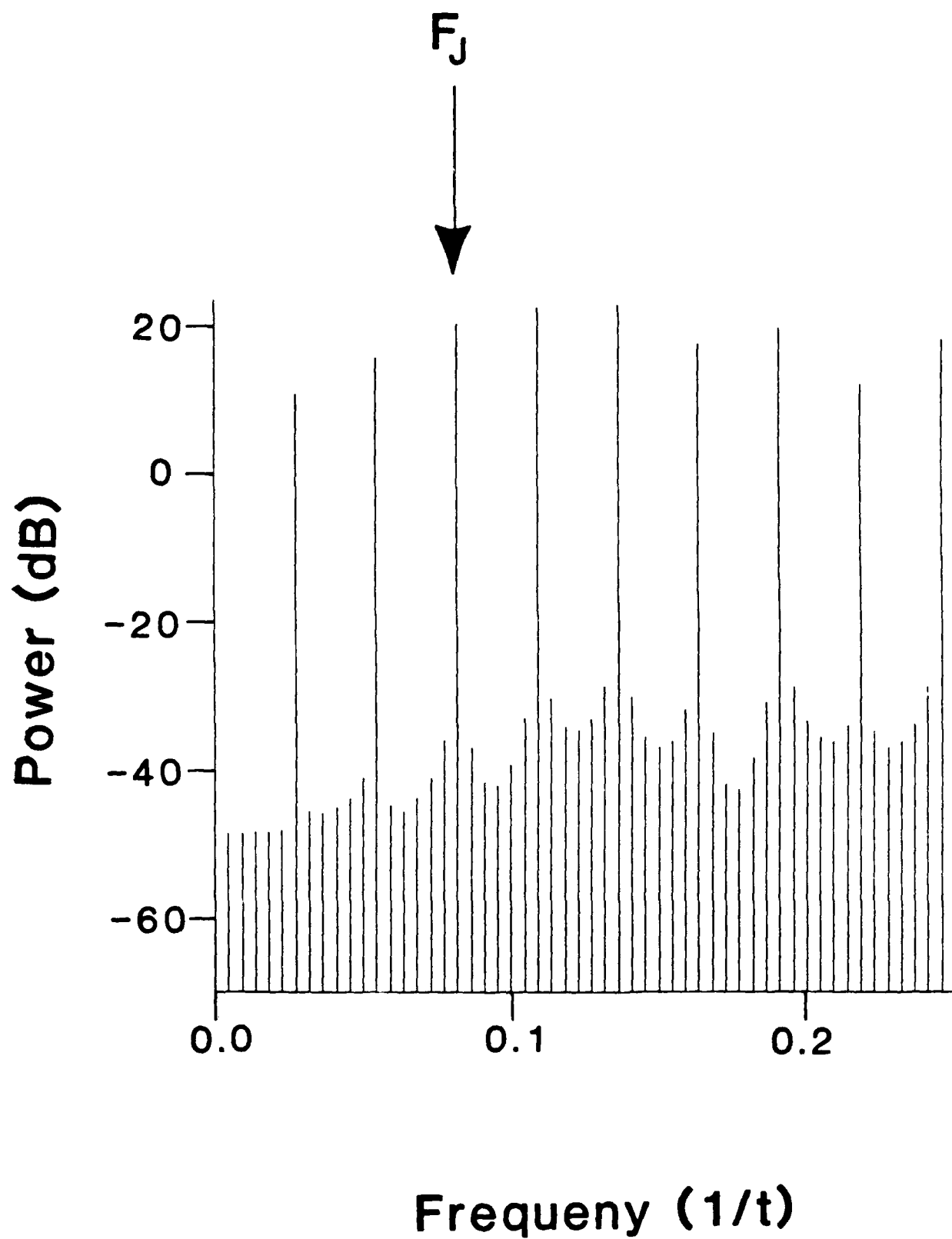


Figure 20: Period-3 window at  $\eta = 0.457$ , power spectrum.



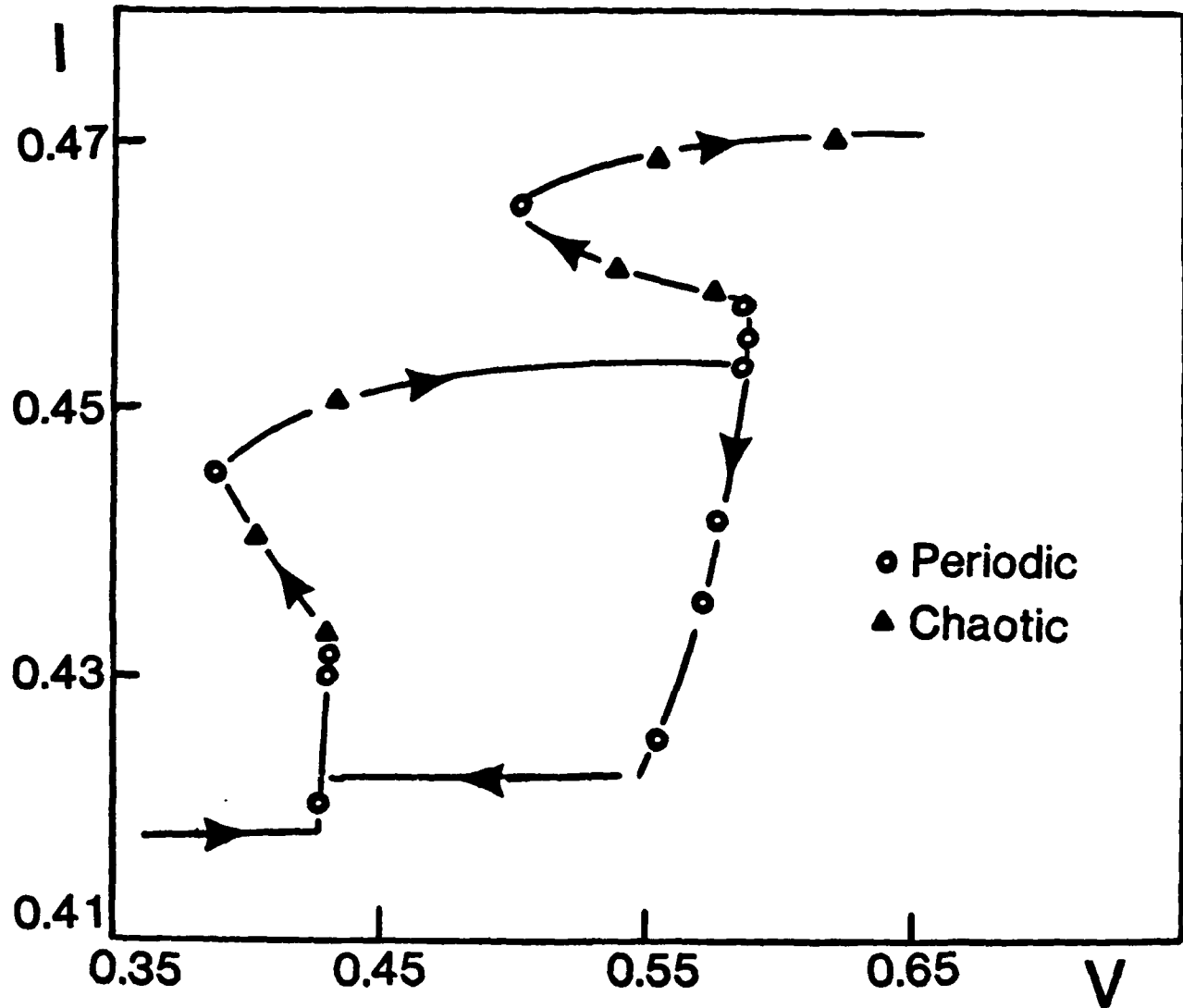


Figure 21: Computed I-V curve for our junction showing negative resistance region for both steps.

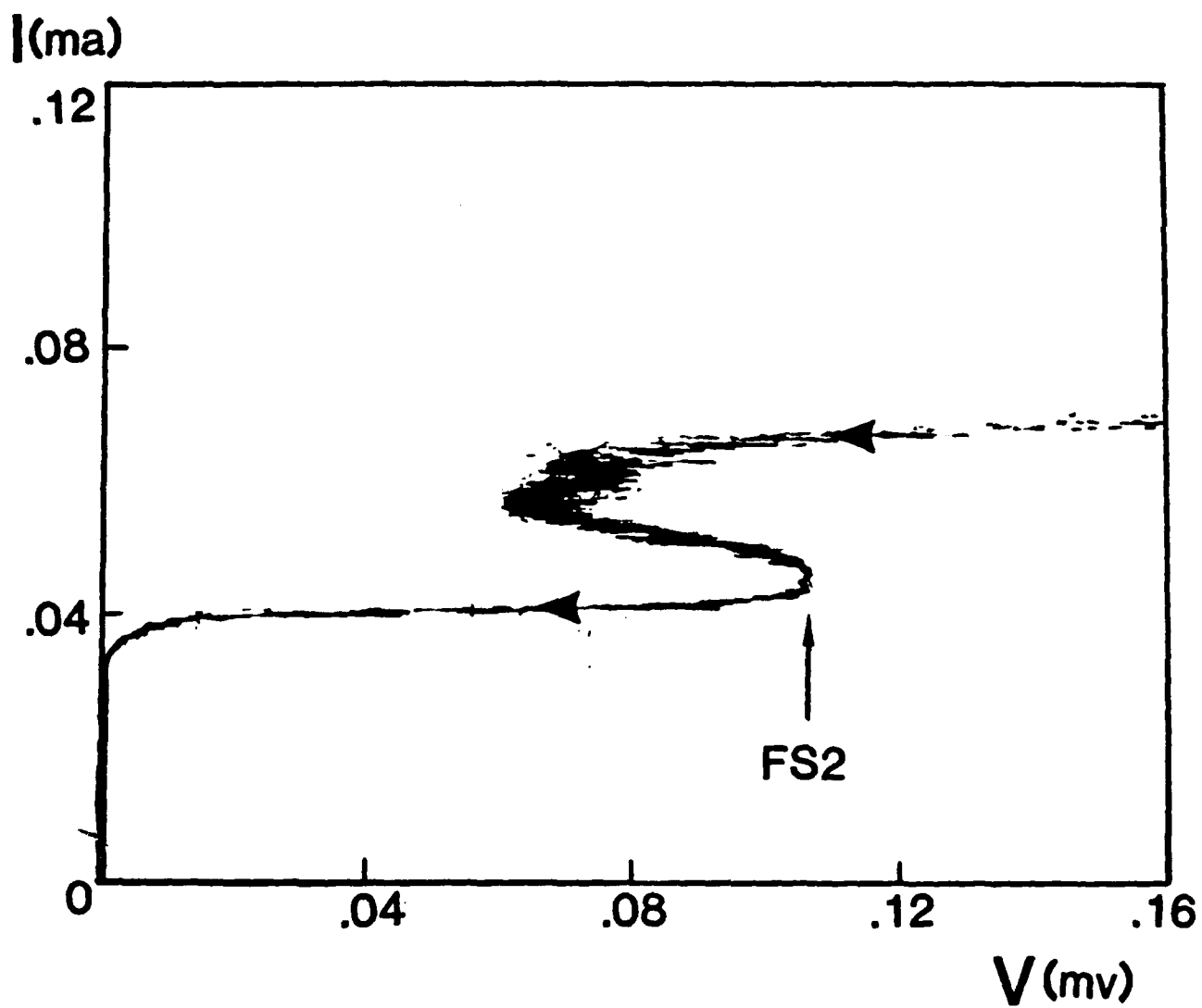


Figure 22: Measured period doubling bifurcation on a real Nb  $\text{Al}_2\text{O}_3$  Nb junction and negative resistance region.

but it could amplify coherent small signals. As suggested in reference 21 and shown in laser systems<sup>22</sup>, small signal detection and amplification can occur near the period-doubling bifurcation. The presence of a negative resistance region in our experimental results on long junctions supports this. A small signal near  $f_0/2$  will result in a large response of the system at the signal frequency. We are presently getting ready to test this. It is particularly important for parametric amplification with Josephson junctions. Actually this non-linear system is analogous to a parametric amplifier operated in the 3-photon mode where the signal is one half of the pump frequency. Actually, studies of Josephson parametric amplifiers<sup>23, 24</sup> show good gain but noisy characteristics. The latter may be due to ideas and results presented here.

## 2.9 Studies of Fluctuations in NbN long Junctions

We have extended our studies of fluctuations and noise to NbN-MgO-NbN long junctions. This material has excellent characteristics for devices and its characteristic lengths are very different from the Nb ones. The coherence length is approximately  $40\text{\AA}$  while in niobium it is of order  $400\text{\AA}$ . This makes the physics quite interesting as the defects will play a stronger role. To get the best sensitivity for measuring fluctuations, a SQUID sensor was used with special coupling to the long junction. Figure 23 shows the schematic of the measuring system used successfully by us for long junctions. Since the junctions have to be current biased at fixed voltage regions, the coupling to the SQUID could not be of the conventional type with a superconducting flux transformer. A resistive network had to be incorporated in the coupling network to keep the currents at reasonable levels. Since room temperature leads go directly to the long junction for biasing, large amount of filtering had to be used so as not to interfere with the SQUID. Typical current steps on the I-V curve are shown in Figure 24; next to this curve, the figure displays some of the large noise

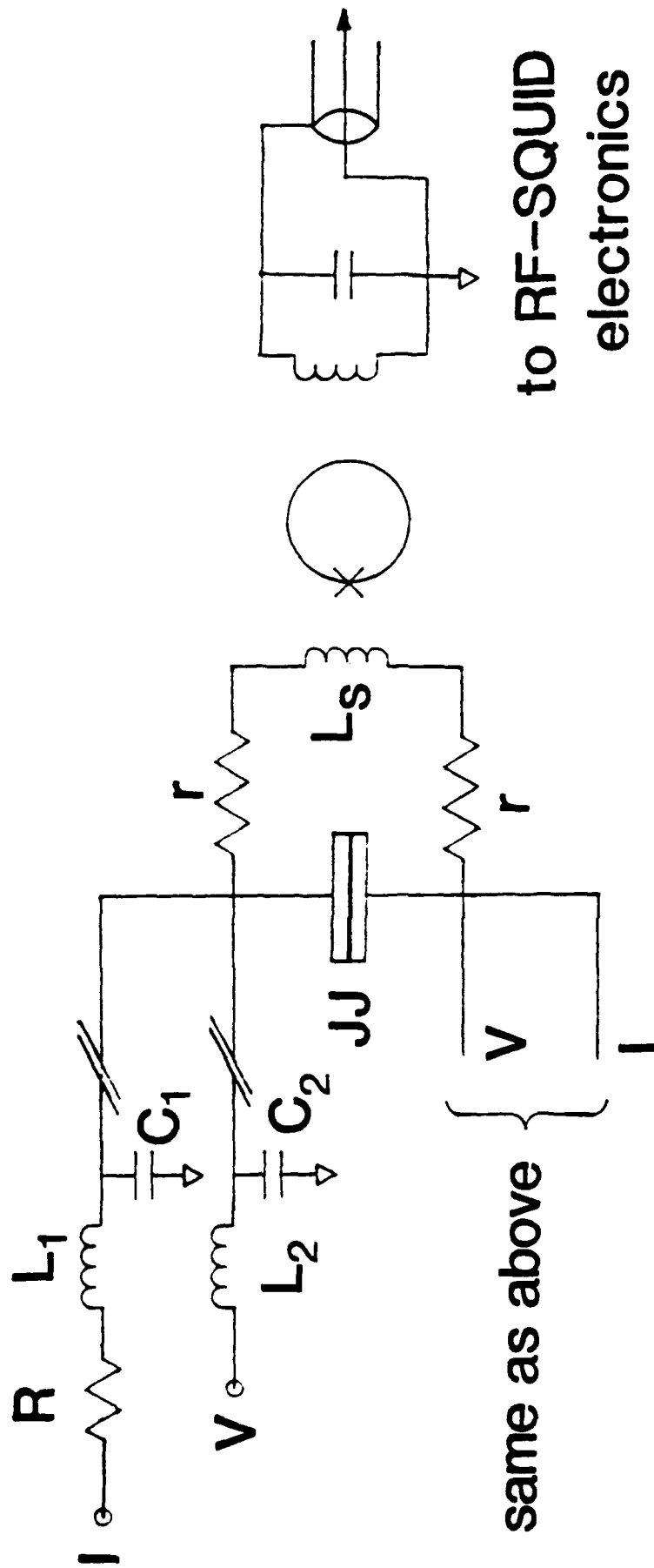


Figure 23: Circuit for measuring junction noise using SQUID magnetometer.

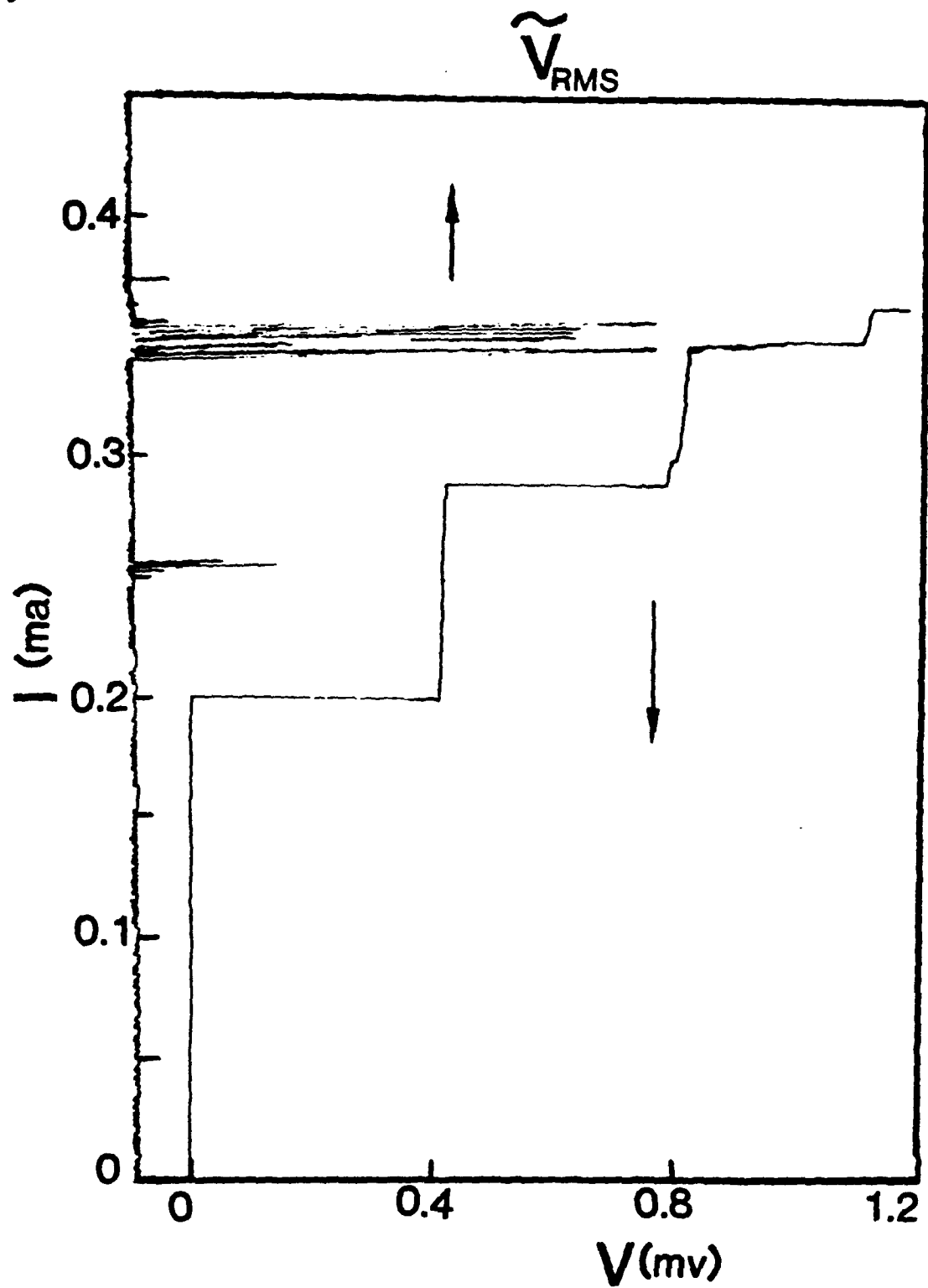


Figure 24: Current steps on I-V curve for NbN-MgO-NbN long junction. Top part shows fluctuations at critical parts.

fluctuations observed at various bias conditions. This illustrates, as before, how the non-linear characteristics of the junction produce instabilities. Details of this experiment will be written up in a forthcoming publication. After the present experiments in the 4.2K to 1.5K range, the sample will be cooled in a dilution refrigerator for studies of quantum mechanical tunneling of fluxons and the effects of dissipation.

### 3. PUBLICATIONS AND CONFERENCE PRESENTATIONS

- Fluxon Fluctuations in Long Josephson Junctions, B.S. Han, B. Lee, O.G. Symko, and D.J. Zheng, Proc. 18th Int. Conf. on Low Temp. Phys., Jap. Journ. of Applied Physics, 26, Suppl. 26-3, 1551 (1987).
- Effect of Injected Current Geometry on Gain of Long Josephson Junction, B. Lee, O.G. Symko, and D.J. Zheng, Proc. of 1987 Intern. Superc. Electronics Conference (ISEC 1987), P. 204.
- Noise Characteristics and Instabilities of Long Josephson Junctions, B.S. Han, B. Lee, O.G. Symko, W.J. Yeh, and D.J. Zheng, Proc. IEEE, MAG-25, 1989.
- Noise and Fluctuations in Long Josephson Junctions in a Magnetic Field, B.S. Han, B. Lee, O.G. Symko, and D.J. Zheng, to be published.
- Chaos in Long Josephson Junctions Without External R.F. Driving Force, W.J. Yeh, O.G. Symko, and D.J. Zheng, in preparation.
- Period-Doubling in a Perturbed Sine-Gordon System, a Long Josephson Junction, D.J. Zheng, W.J. Yeh, and O.G. Symko, to be published.
- Josephson Junction Noise Measurement Using a SQUID Magnetometer, L. Baselgia, O.G. Symko, W.J. Yeh, and D.J. Zheng, APS Solid State Meeting in St. Louis, 1989.

#### 4. PERSONNEL

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 B.S. Han, Visiting Scientist, 1986-87.  
 Bill Lee, Graduate Student, Ph.D. 1986.  
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 Orest G. Symko, Principal Investigator

#### 5. FUTURE TRENDS

We have shown that a highly non-linear system such as the long Josephson junction has regions of instabilities and chaotic behavior separated from periodic regions. This system is also rich in phenomena dealing with noise and fluctuations. Such studies are useful for a fundamental understanding of these effects and also from a practical point of view. For example, our studies show where this type of device should be biased in order to avoid excessive noise. Such results are of immediate interest to applications of the long Josephson junction to microwave oscillators and analog amplifiers.

The oscillators have applications to microwave receivers, fundamental studies of single-electron Josephson junctions, broadband devices, and basic studies of superconductivity. They are especially useful because of the relatively high power levels and because of the tuning feature over a wide range of frequencies. There is a possibility that the noise, under certain conditions, can even be reduced<sup>20</sup> by squeezing at some phase near the period-doubling bifurcation.

The non-linear behavior of fluxons studied in this project is of importance to the field of non-linear phenomena and to high  $T_c$  superconducting materials. Defects in these materials create arrays of naturally occurring long Josephson junctions. Indeed, some of the microwave losses in the new materials are attributed to Josephson fluxons propagating according to equation 4.

The most interesting trend of this research will be when the devices are reduced in size. The behavior of fluxons will then lead to a variety of interesting effects as well as practical applications in high speed electronics. Fluxon devices may interface quite well with the possible new devices based on single electron transistors<sup>25</sup>.



### APPENDIX

In this section all of the papers that have resulted from this work are attached. Because some of them are being written now and/or being analyzed, there will be a slight delay in having this section completed. The entire Appendix section will be submitted separately within a few weeks.

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